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ON THE APPLICABILITY OF THE DISSOCIATION THEORY TO THE ELECTROLYSIS OF AQUEOUS SOLUTIONS CONTAINING TWO ELECTROLYTES WITH A COMMON ION.

By J. G. MacGregor.

SCHRADER,[1] Hopfgartner,[2] and Hoffmeister[3] have recently published valuable researches on the electrolysis of aqueous solutions containing two electrolytes with a common ion, determining the amounts of the distinctive ions, in each case, transferred by the current across a section of the cell. The main object of these researches was the determination of the effect produced on the state of ionization of an electrolyte in solution by the addition of different quantities of a second electrolyte; and although electrolytic observations are subject to so large an experimental error that ionization coefficients determined by means of them cannot be expected to have any high degree of accuracy,[4] it was found possible in some cases to show that the effect produced on the state of ionization of

[1] Zur Elektrolyse von Gemischen, Inaug. Diss., Berlin, 1897; also Ztschr. f. Elektrochemie, 3, 498.

[2] Ztschr. f. phys. Chemie, 25, 115, 1898.

[3] Ibid., 27, 345, 1898.

[4] Hoffmeister, for example, determined the values of the equilibrium constant (*i. e.*, the ratio of the frequency of the recombination of free ions to the frequency of the dissociation of molecules) for $AgNO_3$ in a series of solutions containing also HNO_3, and found them to be 1.12, 1.30, 1.45, 1.04, 1.57, the dilutions of the solutions with respect to $AgNO_3$ ranging from 1.11 to 10 liters per grm.-mol. According to the dissociation theory, the above ratio should not vary with dilution.

one electrolyte by the presence of another in the same solution was of the kind required by the dissociation theory.

The observations referred to may be employed to test the applicability of this theory, in a more satisfactory way, by determining the ionization coefficients in the complex solutions independently of the electrolytic results, finding by calculation what the relative amounts of the distinctive ions transferred by the current ought to be according to the theory, and comparing the calculated with the observed values. I have applied this test in the case of all the solutions referred to above for which the requisite data are available, viz. Schrader's containing KI and KCl, and H_2SO_4 and $CuSO_4$, and Hopfgartner's containing NaCl and HCl, and $BaCl_2$ and HCl. The results are given below.

According to the dissociation theory, if a solution contain N_1, N_2 grm.-equivalents per liter of two electrolytes, 1 and 2, having a common ion, if a_1, a_2 are their ionization coefficients, $\mu_{\infty 1}$, $\mu_{\infty 2}$ their equivalent conductivities at infinite dilution and ν_1, ν_2 the Hittorf's transference numbers for the distinctive ions, the ratio of the number of grm.-equivalents of the distinctive ion of 1 to the number of grm.-equivalents of the distinctive ion of 2, transferred by the current across a section of the cell (we may call this ratio the transference ratio for the distinctive ions), is given by the expression

$$a_1 N_1 \nu_1 \mu_{\infty 1} / a_2 N_2 \nu_2 \mu_{\infty 2},$$

provided the solution is sufficiently dilute.

In applying this expression to the above observations, the only quantities involved whose determination presents any difficulty are the a's and the ν's ; for the N's were of course determined by the observers, and Kohlrausch[1] has determined the μ_∞'s for a temperature which is either the same, or sufficiently nearly the same, as that at which the electrolytic observations were made.[2]

The a's were determined, in the case of each solution considered, from the following equations :

$$(a) \qquad a_1 / V_1 = a_2 / V_2,$$

[1] Wied. Ann., 50, 406, 1893.

[2] Schrader does not give his temperature, which I assume therefore to have been the ordinary temperature of a laboratory, not far from 18° C.

where V_1, V_2 are the regional dilutions of electrolytes 1 and 2, (*i. e.*, the quotients of the volumes[1] of those regions in unit volume of the solution which may be imagined to be occupied by 1 and 2 respectively, by the numbers N_1, N_2 of grm.-equivalents of these electrolytes in unit volume of the solution). This equation expresses the condition of dissociational equilibrium.

$$(b) \qquad N_1V_1 + N_2V_2 = 1,$$

an equation which asserts the equality of the volume of the solution to the sum of the volumes of the regions referred to;

$$(c) \qquad a_1 / V_1 = f_1(V_1), \text{ and } a_2 / V_2 = f_2(V_2),$$

equations expressing the concentration of ions in the regions referred to, and at the temperature of the experiments, as functions of the dilutions of the respective electrolytes in those regions, the functions f_1, f_2 being assumed to be the same as those by which the ionic concentrations of simple solutions of the respective electrolytes would be expressed in terms of their dilution, and being determined empirically in each case from observations of the conductivity of simple solutions, on the assumption that the ionization coefficient for a simple solution is equal to the quotient of its equivalent conductivity by its equivalent conductivity at infinite dilution.

These equations were solved graphically. Curves were plotted for simple solutions of both electrolytes with concentrations of ions as abscissæ and dilutions as ordinates, and points were found on these curves having the same abscissa ($a_1 / V_1 = a_2 / V_2$) and having ordinates such as would satisfy the condition (*b*). The common value of a_1 / V_1 and a_2 / V_2 and the corresponding values of V_1 and V_2 were then read off, and a_1 and a_2 were determined by multiplication.[2]

That this method gives closely approximate values of what the ionization coefficients must be taken to be in such solutions, accord-

[1] In the case of solutions which are so dilute that they might be prepared by the mixture of simple solutions of 1 and 2, without the occurrence of any change of volume on mixing, the volumes of these regions will be the volumes of the isohydric simple solutions of 1 and 2, by the mixing of which unit volume of the given complex solution would be produced.

[2] For a discussion of the above equations and the graphical method of solving them, see Trans. Nov. Scot. Inst. Sci., 9, 101.

ing to the dissociation theory, is borne out by the fact that it has been found possible by their aid to calculate within the limits of observational error the conductivity of dilute solutions containing NaCl and KCl,[1] NaCl and HCl,[2] NaCl and $BaCl_2$,[3] K_2SO_4 and Na_2SO_4,[4] and $ZnSO_4$ and $CuSO_4$,[5] and by means of coefficients determined in a similar way from equations applicable to the case of solutions containing two electrolytes with no common ion, to calculate the conductivity of dilute solutions containing NaCl and K_2SO_4.[6]

It should be noted that the above method assumes the ionization coefficient for a simple solution to be equal to the ratio of the equivalent conductivity to the equivalent conductivity at infinite dilution, and will, therefore, give exact values only at extreme dilution, and only provided (1) the mode of ionization of each electrolyte in a solution is the same for all molecules and for all concentrations, and (2) association of molecules to form complex molecules does not occur.

The conductivity data used in determining the ionization coefficients were Kohlrausch's[7] for all the electrolytes and McIntosh's, McKay's and Archibald's observations given in the papers just cited. The data were fairly complete for the chlorides and copper sulphate, but meagre for potassium iodide and sulphuric acid.

The relation of the Hittorf transference numbers for the ions of electrolytes in a complex solution, to their values for simple solutions, is not definitely fixed by the dissociation theory as at present developed. It is usual to assume that if the solution is dilute the transference number, ν, for the anion or cation of each electrolyte is the same as in a simple solution of concentration equal to the concentration of the complex solution with respect to such electrolyte (assumption *a*). But if, as this assumption implies, the velocities of the ions of one electrolyte in a sufficiently dilute solution are not influenced directly by the presence of other electrolytes, they must also be regarded as not being influenced by the presence of their own undissociated molecules. It would thus seem to be more in line with the dissociation theory to assume (assumption *b*) that the ν for an elec-

[1] Phil. Mag. (5), 41, 276 (1896).
[2] McIntosh, Ibid. (5), 41, 510 (1896).
[3] McKay, Trans. N. S. Inst. Sci., 9, 321.
[4] Archibald, Ibid., 9, 291.
[5] Archibald, Ibid., 9, 307.
[6] Phil. Mag. (5), 45, 151 (1898).
[7] Wied. Ann., 26, 196 (1885).

trolyte in a dilute complex solution has the same value as it would have in a simple solution with concentration of ions equal to the concentration of the complex solution with respect to the free ions of such electrolyte. For infinitely dilute solutions the two assumptions are indistinguishable; but for moderate dilution the second may be expected to give the better results. I have made the calculations of transference ratio for the various solutions on both assumptions and find that assumption *b* gives results which are in slightly better agreement with the observations than assumption *a*. The results given below are those obtained with assumption *b*.

The values of the transference numbers used in the calculations were obtained by the aid of Hopfgartner's[1] observations of their values for series of simple solutions of NaCl, HCl and $BaCl_2$ and the observations on the other electrolytes given in Kohlrausch's[2] and Fitzpatrick's[3] tables. The ionization coefficients a_1, a_2 of the electrolytes in a solution having been determined, the concentrations a_1N_1, a_2N_2 of the solution with respect to the distinctive ions were found. The concentrations of the simple solutions of 1 and 2 having these concentrations of ions were then determined graphically by means of the dilution—ionic-concentration curves required for finding the a's, and the transference numbers for the proper ions of simple solutions of these concentrations were obtained by graphical treatment of the data just referred to.

Solutions Containing NaCl and HCl.

The only series of solutions for which the data were all quite satisfactory and the number of observations of transference ratio not too few to give a definite result, was Hopfgartner's series containing NaCl and HCl; and fortunately in this case no question can arise as to mode of ionization. The results of the calculations are contained in Table I., which gives the concentrations (in grm.-eq. per liter) of the complex solutions with respect to each electrolyte, the ionization coefficients found by the method referred to above, and the observed and calculated values of the transference ratio

[1] Loc. cit.
[2] Wied. Ann., 50, 406 (1893).
[3] Br. Ass. Report, Nottingham, 1893.

(ratio of number of gramme-ions of sodium (M_{Na}) to number of gramme-ions of hydrogen (M_H), transferred by the current. To test the accuracy of the ionization coefficients, I have calculated the equilibrium constants for NaCl and HCl, respectively, by the formulæ :

$$C_1 = \frac{a_1 N_1 (a_1 N_1 + a_2 N_2)}{(1 - a_1) N_1}, \quad C_2 = \frac{a_2 N_2 (a_1 N_1 + a_2 N_2)}{(1 - a_2) N_2}.$$

The values found are given in the table.

TABLE I.

Solutions containing NaCl and HCl.

Concentration with respect to		Ionization coefficients for		Equilibrium constants for		Transference Ratio — M_{Na}/M_H.			
NaCl	HCl	NaCl	HCl	NaCl	HCl	Observed.	Calculated.	Difference.	Diff. per cent.
0.1	0.9	.6596	.7892	1.504	2.906	.01511	.01230	−.00281	−18.6
.2	.8	.6664	.7909	1.530	2.897	.03142	.02782	−.00360	−11.5
.5	.5	.6734	.7986	1.518	2.918	.1024	.1101	+.0077	+ 7.5
.8	.2	.6741	.8036	1.448	2.864	.5336	.4303	−.1033	−19.4
.9	.1	.6759	.8061	1.437	2.864	.9469	.9610	+.0141	+ 1.5

It will be noticed that the values of the equilibrium constants vary somewhat, the extreme differences being about 6 per cent. for NaCl and 2 per cent. for HCl. When we reflect that any percentage error in the a's is increased two or three times in the C's the agreement is seen to be sufficient to justify considerable confidence in the a's.

The agreement between the calculated and observed values of the transference ratio is quite satisfactory so far as the signs of the differences are concerned, and not very unsatisfactory as regards their magnitude ;[1] but the negative differences are somewhat too large relatively to those of positive sign. If we plot the calculated and observed values respectively against ratio of amount of NaCl to amount of HCl in solution (the solutions had all the same total concentration), the calculated curve is found to be practically a straight

[1] The observed values are means of from two to four observations on the same solutions, the means differing from the most divergent individual values by 15, 15, 21, 9.7 and 5.2 % for the respective solutions.

line. The curve of observed values zigzags about this line. The observations are too few to render it possible to draw in with confidence the most probable smooth curve of observed values; but if the observed values be assumed to be representable by a straight line, this line would be slightly less inclined to the transference ratio axis than the line of calculated values. This relation of the two lines would be produced if the more rapidly moving H ions hurried on the more sluggish Na ions and were in turn retarded by them, an effect which may possibly be produced when the current is conveyed by two sets of distinctive cations. Notwithstanding the slight inclination of the line of observed values to the line of calculated values, the calculated values are clearly within the limits of observational error, these limits being, however, necessarily wide. Consequently, in the case of these solutions, the dissociation theory is shown to satisfy the most severe test, which can at present be applied, with regard to its capability of accounting for the results of electrolysis.

Solutions Containing KI and KCl.

Unfortunately the data as to both conductivity and transference numbers for KI are meagre, and the number of solutions examined by Schrader was but four. Table II. contains the results of the calculations.

Table II.

Solutions Containing KI and KCl.

Concentration with respect to		Ionization coefficients for		Equilibrium constants for		Transference Ratio—M_I/M_{Cl}.			
KI	KCl	KI	KCl	KI	KCl	Observed.	Calculated.	Difference.	Diff. per cent.
.02595	.02571	.9071	.9020	0.466	0.430	.9968	1.0050	+ .0080	+ 0.8
.03442	.04748	.8644	.8583	.449	.427	.7037	.7228	+ .0191	+ 2.7
.03074	.06176	.8700	.8611	.534	.495	.5000	.4968	— .0032	— 0.6
.01992	.03720	.8928	.8822	.423	.379	.4864	.5353	+ .0489	+10.1

The variation in the calculated values of the equilibrium constants is greater than in the former case, as indeed was inevitable considering the small amount of information at command as to the variation of the conductivity of solutions of the iodide with concentration. The agreement is such, however, as to warrant some confidence in the ionization coefficients.

The agreement between calculated and observed values of the transference ratio is also less satisfactory than in the former case, the positive differences being too large relatively to the single negative difference. But, all things considered, it may be said to be at any rate consistent with the calculability of the transference ratio.

SOLUTIONS CONTAINING $BaCl_2$ AND HCl.

The data for these solutions are fairly complete; but there are only three observed values, and the mode of ionization of $BaCl_2$ is, of course, doubtful. Table III. contains the results of the calculations, the equilibrium constants being determined, on the assumption that $BaCl_2$ dissociates into Ba and 2Cl, by the formulæ

$$C_1 = \frac{\alpha_1 N_1 (\alpha_1 N_1 + \alpha_2 N_2)^2}{(1 - \alpha_1) N_1}, \quad C_2 = \frac{\alpha_2 N_2 (\alpha_1 N_1 + \alpha_2 N_2)}{(1 - \alpha_2) N_2},$$

$BaCl_2$ being electrolyte 1.

TABLE III.

Solutions containing $BaCl_2$ and HCl.

Concentration with respect to		Ionization coefficients for		Equilibrium constants for		Transference Ratio — $M_{\frac{1}{2}Ba}/M_H$			
$\frac{1}{2}BaCl_2$	HCl	$BaCl_2$	HCl	$BaCl_2$	HCl	Obs'ved.	Calculated.	Difference.	Diff. per cent.
0.2	0.8	.5409	.7975	0.656	2.939	.0120	.0136	+.0016	+13.3
.5	.5	.5508	.8092	.567	2.884	.0487	.0519	+.0032	+ 6.8
.6667	.3333	.5603	.8115	.529	2.788	.0934	.1041	+.0107	+11.5

The equilibrium constants show considerable variation, the extreme differences being about 25% for $BaCl_2$ and 6% for HCl. This was to be expected; for in simple solutions of $BaCl_2$ these constants vary about six times as rapidly as in the case of solutions of NaCl. Although, therefore, the inevitable errors in the α's have a greater effect on the result in the former than in the latter, some additional source of error would seem to be indicated by this greatly increased variation, possibly the dissociation of $BaCl_2$ to a small extent into BaCl and Cl as ions.

The agreement between the transference ratio values is accordingly not so good as in former cases, the differences, though of the same order of magnitude as before, being all positive. Considering

the wide limits of the observational error, however, and the small number of observations, the result cannot be said to be inconsistent with the calculability of the transference ratio, even on the doubtful assumption of uniformity in the mode of ionization of the barium salt.

Solutions containing H_2SO_4 and $CuSO_4$.

The conductivity data for $CuSO_4$ are good and sufficient; for H_2SO_4 they are good, though meagre. If the solutions really contained only two electrolytes with a common ion, and if each electrolyte had only one mode of ionization, fairly good values of the a's might be expected. But the H_2SO_4 may dissociate into either $2H$ and SO_4 or H and HSO_4. According to Ostwald[1] it may be expected to dissociate in both ways in solutions of moderate dilution, like Schrader's. If so, the acid sulphate of copper $Cu(HSO_4)_2$ may be expected to form by association of free ions. And quite independently of the mode of ionization this salt may form by the association of molecules. That it will form is rendered probable by the fact that the acid sulphates of several metals allied to copper are known. In these solutions, therefore, we have possibly solutions of very great complexity to deal with.

Not knowing what the constitution of the solutions really is, I have assumed the acid to dissociate wholly into $2H$ and SO_4 and the acid salt not to form, this being the only assumption which I could use as a basis of calculation. The expressions used for the equilibrium constants accordingly were:

$$C_1 = \frac{(a_1N_1)^2(a_1N_1 + a_2N_2)}{(1 - a_1)N_1}, \quad C_2 = \frac{a_2N_2(a_1N_1 + a_2N_2)}{2(1 - a_2)N_2},$$

H_2SO_4 being electrolyte 1. Table IV. gives the results of the calculations.

The calculated values of the equilibrium constants show enormous variation, especially in the case of the acid, as was to be expected from the corresponding variation in the case of simple solutions of the acid. Nevertheless observations of the conductivity of the above complex solutions, made by Schrader, seem to show that it is only

[1] Jour. prakt. Chemie (2), 31, 433 (1885).

TABLE IV.

Solutions containing H_2SO_4 and $CuSO_4$.

Concentration with respect to		Ionization coefficients for		Equilibrium constants for		Transference Ratio — $M_H/M_{\frac{1}{2}Cu}$			
$\frac{1}{2}H_2SO_4$	$\frac{1}{2}CuSO_4$	H_2SO_4	$CuSO_4$	H_2SO_4	$CuSO_4$	Observed.	Calculated.	Difference.	Diff. per cent.
.02753	.06618	.5928	.3776	.00098	.0125	4.005	4.597	+ 0.592	+ 14.8
.04168	.10446	.5529	.3318	.00165	.0143	3.677	4.776	+ 1.099	+ 29.9
.05297	.12607	.5591	.3156	.00261	.0160	3.084	5.339	+ 2.255	+ 73.1
.06326	.11087	.5605	.3114	.00317	.0158	3.014	7.437	+ 4.423	+146.8
.09256	.16702	.6240	.2808	.01003	.0204	3.391	9.002	+ 5.611	+165.5
.13753	.24899	.6684	.2445	.0283	.0247	3.285	11.223	+ 7.938	+241.7
.19605	.34673	.6415	.2225	.0457	.0290	2.099	12.430	+10.331	+492.1
.07217	.07135	.5544	.3276	.00316	.0154	10.417	12.309	+ 1.892	+ 18.2
.15453	.14727	.6486	.2525	.0254	.0232	3.749	19.639	+15.890	+423.8
.18043	.14830	.6704	.2349	.0384	.0240	5.410	25.233	+19.823	+366.4

the stronger of his solutions whose conductivity is non-calculable, that the above ionization coefficients may thus be regarded as roughly approximate mean values, as it were, and consequently that the large transference ratio differences are probably for the most part due to some other source of error. It may, therefore, be well to ask what effect the formation of the acid sulphate would probably have on these differences if the mode of ionization were as assumed above.

The disagreement between observed and calculated values of the transference ratio, would be due, on the assumption of the formation of acid sulphate, to errors in both the observed and the calculated values. Schrader regarded the difference between the concentration, with respect to free acid, of the portion of a solution surrounding the anode at the beginning and end of each experiment, as equal to the number of gramme-ions of hydrogen transferred by the current. But as he used copper electrodes and found no development of gas at the anode, each gramme-equivalent of the acid salt electrolyzed would produce a gramme-equivalent of free acid at the anode. Also, as the solution in the neighborhood of the anode was continually changing in constitution during an experiment, some acid salt might dissociate non-electrolytically into free acid and neutral salt. Hence his estimate of the hydrogen trans-

ferred would be too small by the number (n) of grm.-equivalents of acid salt electrolyzed and the number (m) non-electrolytically dissociated. His values of the transference ratio would therefore be too small by amounts increasing with $n + m$.

The calculated values would be in error also; for in the expression for the transference ratio, N_1 and N_2 would be too great by N_3, the number of gramme-equivalents of acid sulphate formed per liter, and there would have to be an additional term in the denominator, $+ a_3 N_3 \nu_3 \mu_{\infty 3}$. It is perhaps unnecessary to inquire[1] into the relative magnitudes of a_3, ν_3, and the new and old values of a_1, a_2, ν_1 and ν_2. For it will be sufficiently obvious that the calculated values of the transference ratio must be greater than they would be if the presence of acid salt in the solutions were taken into account, and greater by amounts increasing with N_3.

The calculated values being too great by amounts increasing with N_3 and the observed values too small by amounts increasing with $n + m$ (n, of course, increases with N_3), the differences between them will be large and will increase very rapidly with N_3, provided m is positive. If m is negative they will, *ceteris paribus*, increase less rapidly, and if m, being negative, becomes large, they may increase slowly or even begin to decrease. Obviously, if the differences be expressed as percentages of the observed values, the errors in the observed values will have double importance given them, and in cases in which m is negative and sufficiently large the differences will reach and pass a maximum value at an earlier stage. Now, as Schrader used copper electrodes, the concentration of his solutions, with respect to $CuSO_4$, near the anode, must have been increasing during each experiment. In the solutions of the first division of Table IV., therefore, which contain less acid than salt, some of the acid salt present at the beginning would dissociate during the experiment, non-electrolytically, into acid and neutral salt, *i. e.*, m would be positive. In this division, therefore, both the absolute and the percentage differences may be expected to increase rapidly throughout, as the table shows they do. In the solutions of the second division which contain more acid than salt, incre-

[1] This inquiry is made in a communication to the Royal Society of Canada at its last meeting, of which the present paper is a shortened account.

ment of the concentration with respect to salt would probably result in the formation of acid salt, *i. e.*, m would be negative. In the last solution which contains a good deal more acid than salt, m would be negative and pretty large. The fact, therefore, brought out in the table that in the second division the percentage differences increase rapidly at first and finally reach and pass a maximum, becomes quite intelligible.

The assumption of the existence of acid salt in the solutions enables us, therefore, to account in a very complete manner for both the sign and the relative magnitude of the differences between the observed and calculated values of the transference ratio.

In the case of the solutions of H_2SO_4 and $CuSO_4$ therefore, we have neither sufficient knowledge of the constitution of the solutions nor sufficient data, to test the applicability of the dissociation theory. But by applying the theory to a tentative conception of their constitution which is known on other grounds to be probably inadequate in one important point, we are able to show that the differences between calculated and observed values are such as the neglect of that point would produce.

DALHOUSIE COLLEGE, HALIFAX, N. S.

ON THE PERIOD AND LOGARITHMIC DECREMENT OF A CONTINUOUSLY VIBRATING WIRE.

By Joseph O. Thompson.

THE experiments described in the following article originated from a desire to study more minutely the phenomena to which Lord Kelvin[1] called attention in the following words:

"Experimental exercises performed by students in the physical laboratory of the University of Glasgow, during the session 1864–65, brought to light some very remarkable and interesting results, proving a loss of energy in elastic vibrators (sometimes as much as two or three per cent. of energy lost in the course of a single vibration is one direction) incomparably greater than anything that could be due to imperfections in their elasticity, and showing also a very remarkable fatigue of elasticity, according to which a wire which has been kept vibrating for several hours or days through a certain range came to rest much quicker when left to itself than when set in vibration after it had been at rest for several days and then immediately left to itself. Thus it was found that the rates of subsidence of the vibrations of the several wires experimented on were generally much less rapid on the Monday mornings, when they had been at rest since the previous Friday, than on other days of the week, or than after several series of experiments that had been made on a Monday."

It is well to quote also the following statement of the object of an experiment:

"Two equal and similar wires, with equal and similar vibrators, one of them kept as continually as possible in a state of vibration, from day to day; the other kept at rest, except when vibrated in an experiment once a day (to test the effect of continued vibration on the viscosity of a metal)."

[1] Math. and Phys. Papers, Vol. III., p. 22.

One of the results of this experiment is given in the note of May 27 :[1] "No. 1 has been kept at rest from May 17, while No. 2 has been kept oscillating more or less every day till yesterday, May 26, when both were oscillated, with the following results :

	Time per Vibration.
No. 1 subsided from 20 to 10 after 100 vibrations in 242 sec.	2.42
No. 2 subsided from 20 to 10 after 44 or 45 vibrations.	2.495"

Thus it seems that Lord Kelvin attributed the increased period and the increased logarithmic decrement of No. 2 to the fact that it had been kept in such constant vibration. I also had no doubt that both the above-mentioned quantities were functions of the time the wire had been kept in vibration, and I accordingly went to work to investigate the nature of those functions.

The variations in period of vibration, since they so accurately give a measure of the variations in the rigidity modulus of the wire, were first studied, and since the modulus of elasticity is such a complicated function, it was evidently desirable to keep constant the quantities that are ordinarily variable. The photometer-room in the basement of the Physical Laboratory of Amherst College furnished a reasonably constant temperature, the temperature change rarely exceeding half a degree in twenty-four hours. To keep the pendulum in constant vibration day and night, and, what is vastly more important, to keep the amplitude of vibration constant, the following arrangement was used : I took for the disc of the torsion pendulum a brass plate 24 cm. in diameter and weighing 10½ lbs. Two iron rods ¼" in diameter were screwed into diametrically opposite holes near the edge of the disc ; these rods rose a little way, and then were bent into arcs parallel to the edge of the disc, as indicated in the accompanying diagram. Above the disc and not connected with it were two solenoids wound on lead pipes bent into arcs similar to the iron rods just mentioned. The iron rods served as cores of the solenoids. Hanging from the bottom of the disc near the edge was a light strip of brass carrying on its lower end a platinum tip and also a glass tip. When the disc swung one way the platinum tip made a brief contact with a platinum plate below; this

[1] Vol. III., p. 26.

sent an electric current up the brass strip, across the disc to the center, down the wire to a mercury cup, through the solenoids. The iron cores were sucked in, and thus a small impulse was given to the vibrating disc at the moment it was passing its equilibrium

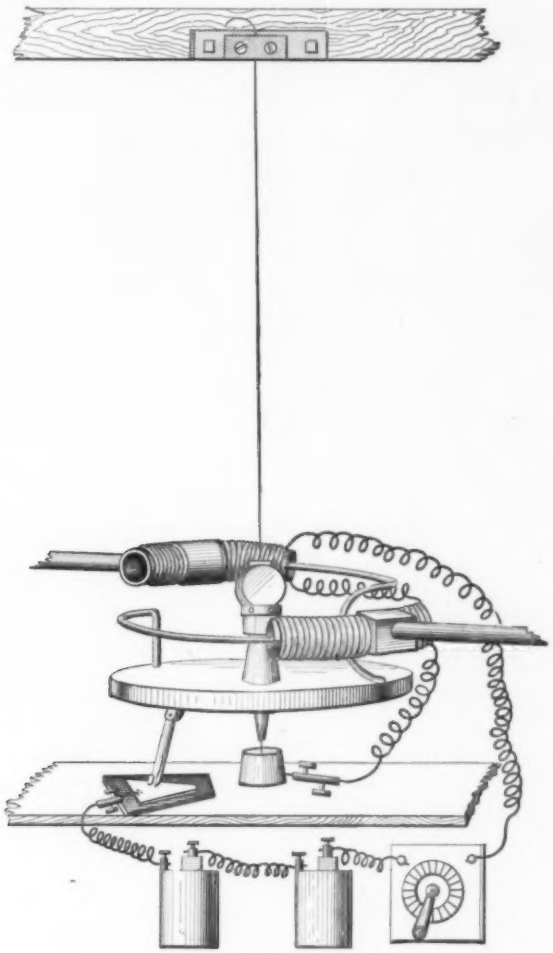

Fig. 1.

position. When the disc swung the other way the insulating glass tip made contact with the platinum plate and no current passed. Thus the plate could be kept vibrating any number of hours or days, and with an amplitude nearly constant. By proper regulation of the

current any arc of vibration from 10° to 250° could be maintained for any length of time. An excellent feature of this device is that it automatically varies the impulse according to the needs of the disc. Suppose for instance we wished to maintain a constant arc of vibration of 100° and the current was regulated accordingly, and suppose some temporary disturbance caused the arc to fall to 90°. The time of electrical contact and consequently the impulse would thereby be increased and the pendulum would soon regain its former amplitude. In most of my experiments half an ampère sufficed to maintain a total arc of vibration of 180°, and the time of electrical contact was about $\frac{1}{40}$ the entire period of vibration. Hence the drain on the storage battery was exceedingly slight.

Since copper wires were used in Lord Kelvin's experiments I was especially careful in studying the behavior of that metal. Experiments were also made with wires of brass, aluminium and silver. No matter what metal was used, no matter whether the arc of vibration was as small as 20° or as high as 200°, no matter whether the wire was long or short, thick or thin (provided of course the breaking strength of the wire was at least twice the weight of the disc) the result was uniformly the same, namely, *that when temperature and amplitude of vibration remained constant, the period of vibration was a constant quantity.* In one experiment with a copper wire the period obtained after fifty hours of continuous vibration through an average arc of 180° was found to be just what it was in the first two minutes. Gauss' method of timing was used and the degree of accuracy aimed at and attained was about $\frac{1}{2000}$.

The following table shows the results of May 16th, a day on which the temperature at the beginning of the experiment in the

Time.		Temperature.	Period.
h.	min.	deg. cent.	sec.
9	20	17.0	21.564
9	58	17.2	21.568
11	30	17.4	21.588
11	49	17.4	21.588
1	57	17.0	21.575
3	09	17.0	21.571
4	12	17.0	21.565

morning was the same as at the end of the experiment in the afternoon. The wire was of copper, 37.7 cm. long and .081 cm. in diameter. The arc was 180°.

The figures for the period indicate, though they are not accurate enough to prove, what I suspect to be the truth, namely, that the period changes with the temperature, not instantly, but in a somewhat sluggish manner.

The Logarithmic Decrement.

Having obtained the above evidence that the continuance of the vibrations had no effect on the period it seemed best to make a still more careful investigation of the effect of this continued vibration on the logarithmic decrement. Since I doubted whether the ordinary method of measuring the decrement would be sufficiently accurate to give any conclusive results, and since the small amplitudes ordinarily employed seemed unlikely to produce any measureable variation in the decrement, I made the following modification of the ordinary method of measuring the logarithmic decrement.

Two silvered mirrors were mounted 8 mm. apart and were carefully adjusted so that they were parallel within 1′. These two mirrors were placed on the vibrator, one on each side of the wire, and instead of being parallel to the telescope scale were made perpendicular to it at its central point. Then when the vibrator was turned 90° from its position of equilibrium readings could be made near the center of the scale; and, just as it is unnecessary to change tangents to arcs when the amplitude does not exceed 2 or 3 degrees, so in our case it is unnecessary to make any correction when the amplitude varies only a few degrees from 90°. Ordinarily in getting the decrement I allowed the total arc of vibration to fall from about 185° to 175°, and then the electrical driving device would be applied to bring the amplitude up to its original value. It must be remembered that in the case of all decrements mentioned later the mean arc was about 180°.

To show exactly how the method was applied I give the following figures, referring to a set of observations made on a copper wire, the one already mentioned above:

The distance from mirror to scale was 98.66 cm.; the circum-

ference of the circle of this radius is 619.9 cm., but I have called it 620 cm. in the computation.

Scale readings.	Differences.	Double arcs.	Logarithms.
cm.		cm.	
42.60			
54.70	+12.1	632.1	2.800786
47.80	+ 6.9	626.9	2.797198
49.55	+ 1.75	621.75	2.793616
52.88	− 3.33	616.67	2.790053
44.50	− 8.38	611.62	2.786481
57.90	−13.40	606.60	2.782902

1st log. — 4th log. = .010733
2d " — 5th " = .010717
3d " — 6th " = .010714

Mean .010721

.010721 ÷ 3 = .003574 = Log. Dec.

A careful inspection of the above figures will show the exceptional accuracy of the method. The period of the pendulum was 21.51 sec., and by making the vibrations slower, greater accuracy in getting the scale-readings could, of course, be secured. When the arc was so great, over 180°, a quick eye was needed in order to be sure of the fractions of a mm.; but if I had made the period greater there would have been less likelihood of detecting variations either in period or in decrement.

The following measurements will be interesting on account of the fact that the wire was kept vibrating through an average arc of 180°

Time.	Temperature.	Log. Dec.
July 5.	°	
2.27 P. M.	23	.00364
2.33	23.1	370
2.39	"	365
2.49	23.2	365
3.09	"	363
3.53	"	367
4.37	23.3	367
5.33	"	365
9.21	21.9	358
July 6.		
8.53 A. M.	20.6	367

for 50 consecutive hours. The wire was the copper wire already mentioned, and had a breaking strength of 32 lbs. The disc weighed 10.5 lbs. Since the experiments of Schmidt[1] show that the elastic after-effect has a very marked influence on the logarithmic decrement no measurements were made until the wire had been stretched over a week by its load.

Thus after 18 hours of continuous vibration the decrement was only a trifle higher than the average for the first afternoon. The second decrement, .00370, owes its large value to the fact that just before it was measured the vibrating disc was accidentally given an amplitude about two degrees larger than it had at the start. The small value, .00358, I cannot with certainty account for.

Since it seemed clear that nothing of value would be gained by trying to keep the amplitude constant, I varied the experiment by increasing the arc to about 250°, noting the increased decrement and the gradual recovery.

Time.	Temperature.	Log. Dec.
July 6.		
9.10 A.M.	20.9	.00372
9.16	21.1	370
11.45	21.4	365
2.27 P.M.	21.3	358

Then the largest possible arc permitted by the apparatus, 250°, was given and the three decrements measured in the next 15 minutes were .00367, .00367 and .00371. On my return at 9 o'clock in the evening the arc was for some reason reduced to about 120° and the decrement, as was to be expected, was lower than any hitherto found, .00346. It became .000363 after the disc had been given its greatest possible amplitude.

The next day, July 7th, at 9:40 A. M., the arc was again down to about 120° and the decrement was found to be .00343. Temperature was 20°.8. At two in the afternoon the temperature was 21°.4 and the decrement .00357. After the amplitude was given its greatest possible value the decrement was found to be .00367.

Between these two last named values of the decrement lies the

[1] Schmidt, Annalen der Phys. n. F., Bd. II.

value obtained 48 hours before at the very beginning of the experiment, namely, .00364. Considering the foregoing evidence and the fact that the smallest decrement, .00343, means an arc ratio 1.0079 and the largest decrement, .00372, a ratio 1.0086, and considering the further fact that the former decrement was obtained after the arc had for some time been 120° while the arc was 250° shortly before the largest decrement was measured, it seems probable to me that for constant temperature and constant amplitude the logarithmic decrement is a constant.

Finally this experiment gives corroborative evidence as to the constancy of the period. The very first measurement of the period made in the early part of the afternoon of July 5th gave as a result 21.496 sec. at the temperature 23°.2. The very last measurement made July 7th after the disc had been vibrating for over two days through an average arc of 180° was identically the same, 21.496 sec. The temperature was 22°. The reason why this value of the period differs from the values given above is that those values were obtained as the pendulum was electrically driven, while this lower value was obtained when the pendulum was swinging absolutely free, being kept in vibration by an occasional tap of the pencil. This tapping method was the more convenient while I was giving my attention chiefly to the logarithmic decrement. The above figures indicate that the electrical impulse was given not at the instant the disc was passing its equilibrium position but a little later.

EXPERIMENTS WITH INCREASED AMPLITUDES.

After having reached the conclusion that under conditions of constant temperature and constant amplitude both the period and decrement are constant, it seemed very desirable to find out if possible how Lord Kelvin could have obtained results which apparently differ so much from the results I have mentioned.

I accordingly removed the solenoids so that the arc of vibration might be increased up to any desired amount, and then I increased the arc from its former value, 180°, to about 360°. With this large arc the period was 21.66 sec. and twenty minutes later, when the arc averaged about 200°, the period was exactly the same, 21.66 sec.

The decrement, as was anticipated, showed a large increase and a surprisingly rapid recovery, as is indicated in the following table:

Time.	Temperature.	Log. Dec.
July 7.		
3.58 P. M.	22.1	.00506
4.02	"	486
4.09	"	470
4.38	22.2	445

These decrements, as in all the preceding cases, were measured while the arc was decreasing from about 185° to 175°.

At 4.45 the period for an arc of 180° was 21.52 sec., thus showing a prompt recovery.

At 4.53 after over 50 hours of continuous vibration the disc was brought to rest.

The following day, after some experiments with the same wire, the decrement had recovered to .00391, and the period showed an entire recovery, being 21.496 sec. at 22°.7.

Finally the disc was given an arc of about 360° + 180°, and after the arc had died down to 180°, the period was 21.68 sec. and the decrement .00672, by far the largest value yet found. *Thus was demonstrated the fact that by suitably modifying the antecedent amplitude of vibration any decrement within wide limits and any period within narrow limits might be obtained.*

Hence such results as those noticed by Lord Kelvin may be produced by giving the vibrating disc a very large amplitude. It seems to me not improbable that at times very large amplitudes were given to his disc, for his words "as continually as possible" imply that no device was used in order to keep up the vibrations. And if all the impulses had to be given by hand, and by students' hands, some of those impulses were probably pretty large, assuming of course that there are points of resemblance between American and Scottish students.

Experiments with Metals other than Copper.

Since the experiments cited at the beginning of this article were made with copper wire I naturally gave more attention to that metal than to any other. It was very desirable, however, that corroborative experiments be made with other metals and accordingly a care-

ful study was made of the behavior of wires of aluminium, silver and brass. The results obtained with aluminium were entirely similar to those obtained with copper. The aluminium wire was 96 cm. long, .108 cm. in diameter, and at the beginning of the experiments, July 11th, had a period of 20.270 sec. and a decrement of .00670 at 19°.6. After over 10 hours of continuous vibration, the temperature being 21°, the period was 20.285 sec. and the decrement .00724. This large decrement is due to the fact that 45 minutes before this I had given the disc an arc of vibration of 360°. The decrement thereby produced was .00853, and the decrements measured in the next 45 minutes (.00764, .00739, .00724) indicated that in an hour more the decrement would have returned nearly to its original value. These large decrements mean of course a much more rapid dissipation of energy than what took place in the copper wire. The current used in keeping the aluminium wire in vibration was therefore considerably greater, being about .75 ampère, whereas in the case of the copper wire a current of .5 ampère sufficed.

The silver wire corroborated the testimony of the other wires even though it had to bear a load unreasonably heavy. The breaking strength was 18 pounds, and the disc weighed 10.5 pounds. No thicker wire was readily obtainable and accordingly experiments were made with this, although it was anticipated that the inevitable stretching of the wire would render the results of little value. The decrement was so large that in order to get four consecutive readings on the telescope scale I had to use a period of vibration considerably larger than any hitherto used.

On the morning of July 13th, the period was 30.65 sec. at 19°.2, the next day in the morning the period was 30.73 sec. at 19°.6, and in the afternoon, after 32 hours of continuous vibration through an arc of about 180°, the period was 30.75 sec. at 19°.85. Just how much of this increase was due to the rise in temperature I do not know; but at any rate, considering the length of the wire was only 29 cm. and the average arc was 180°, and the very important fact that the load was considerably more than half the breaking strength, this maltreated wire did remarkably well in maintaining a constant period.

Numerous data from which the logarithmic decrement might be

computed were taken; but since a careful inspection showed that they modified in no way the results already mentioned no decrements were computed.

The obvious reason for measuring all periods and decrements only when the arc of vibration was between definite limits, say 185° and 175°, was that by so doing the resistance of the air did not need to be determined, being the same in all measurements.

In conclusion I will repeat that the above experiments seem to me to prove that in any vibrating wire that is not unduly loaded, when temperature and amplitude of vibration are constant, both the period and logarithmic decrement are constant quantities.

PHYSICAL LABORATORY, AMHERST COLLEGE.

A PORTABLE APPARATUS FOR PHOTOGRAPHING CURVES OF TWO VARIABLE CURRENTS SIMULTANEOUSLY.

BY H. J. HOTCHKISS.

THE apparatus was presented at the Buffalo meeting of the American Association for the Advancement of Science in 1896, at which time a brief abstract was given in the *Electrical World*, September 26th. A more complete description has been delayed until a few details of construction were more fully developed.

The non-portable single-curve apparatus from which the present form was developed was described in the PHYSICAL REVIEW, July, 1895.[1] The purpose of this article is to describe the present form of the apparatus and give a few curves without taking up any special experiments performed with it.

The aim in designing was to provide a portable instrument which will give continuous simultaneous records of two variable currents or potentials so as to show their corresponding values at any instant. The variations whether cyclic or noncyclic are recorded as fast as they occur, the special province of the apparatus being in experiments dealing with rapid noncyclic variations. The corresponding values of any number of cyclic curves may be obtained by taking two, then one of these with a third, and so on.

The principal parts are two quick acting galvanometers of the Deprez type with minute moving parts supported by a quartz fiber in the strong field of a permanent magnet. These are mounted one above the other in one end of a case about $6 \times 6\frac{1}{2} \times 18$ inches, shown in Fig. 1. In the other end of the case is a revolving drum

[1] Experiments in which the single-curve apparatus was used are described by F. E. Millis in the PHYSICAL REVIEW, March, 1896, p. 351; and September, 1896, p. 128; and others by F. J. A. McKittrick in Trans. A. I. E. E., Vol. 13, May 20, 1896. Some experiments and considerations leading to the present form were discussed in a thesis by the writer in 1896.

carrying a photographic film. The arrangement of parts inside may be seen by referring to Fig. 6.

Sun light or arc light is directed through two narrow slits in the side of the case upon the very small mirrors of the moving parts, which we will call the needles. From the mirrors the two beams are reflected upon a horizontal slit in the drum-box. As the needles are deflected by the current in the galvanometer coils the spots of light move along the slit and trace the curves on the film which moves rapidly at right angles to the slit. The drum-box may be removed and replaced by a slideway for using plates, as seen in Fig. 3, in which the drum-box is used simply as a support or one end of the case.

The needle is a very small piece of soft iron mounted on one side of a quartz fiber with a mirror, about one-third as large as the iron, on the other side. The fiber is supported vertically in the instrument. A medium size for the iron is: 1 mm. wide, 1.9 mm. long with rounded ends, and .06 mm. thick. The long dimension is vertical along the fiber so as to make the moment of inertia small and the frequency of natural vibration high.

The fiber is supported at both ends on a quartz bow or rod about 1 mm. in diameter and 1 cm., or less, in length. The bow is then mounted in a groove, cut lengthwise in the side of a $\frac{5}{32}$-inch rod, about 4.5 cm. long, so that the fiber lies in the axis of the rod as shown full size in Fig. 5.

Fig. 5.

The quartz fibers are mounted on quartz in order to avoid the effects of temperature changes as far as possible. The increase of rigidity[1] with rise of temperature, being only about .01 of one per cent. per degree centigrade, is negligible.

Needles have been made having a natural frequency of over 8,500 double vibrations per second in the comparatively weak field of a permanent magnet weighing about 1.3 kilos. With a strong

[1] S. J. Barnett, PHYSICAL REVIEW, Feb. 1898, p. 114. R. Threlfall, on Laboratory Arts, p. 218.

field and a stiff fiber a frequency of 10,000 double vibrations per second could be obtained. One needle was broken before a reliable test was made, but an approximate measurement gave a frequency of about 9,300.

The frequencies of the needles used in taking the curves shown in Figs. 7–9 and 11–13 are about 5,300 for the voltmeter and 4,200 for the ammeter. The measurements of frequency were made by deflecting the needle by a direct current, then while the plate is before the slit and moving at a speed of about 382 cms. per second the electro-motive force impressed upon the coils is instantly removed by short-circuiting them by means of an automatic mercury switch, leaving the needle free to vibrate due to its inertia. The damping and moment of inertia are small, allowing the needle to respond promptly to variations of current. If very abrupt changes of current cause the needle to vibrate, the mean line gives the true value. Although the appearance of such curves is bad, they are not rendered useless by the vibration.

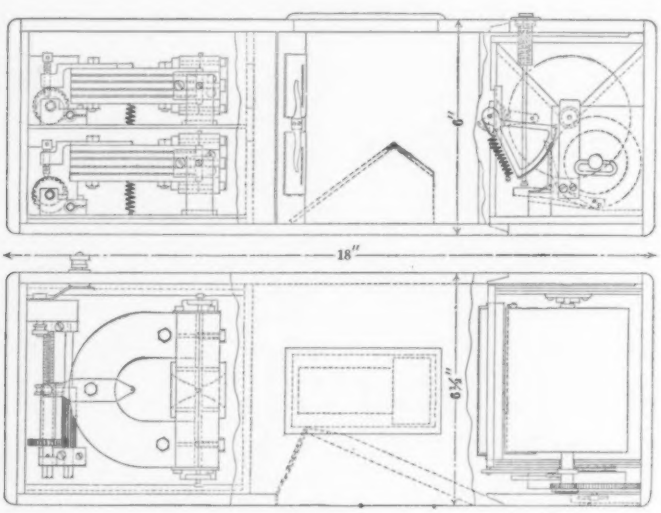

Fig. 6.

The needles are interchangeable so that those of different frequencies and degrees of sensitiveness may be used for different purposes. The galvanometer coils are likewise interchangeable and may also be connected either in series or parallel. In the galvanometers

shown in Fig. 2, one has about 35 turns of number 16 wire in each coil and the other 600 turns of number 30 wire B. & S. gauge, the outer diameter of the coils being 2.5 cms. and the resistances .07 and 17.4 ohms respectively, when each pair is connected in series.

The parts of the galvanometers may be seen in Figs. 2 and 6. The needle rod, coils, and soft-iron pole pieces[1] are held in position between the magnet poles by a two-part hard rubber block which in turn is supported on a base so as to move about a horizontal and a vertical axis through the needle for the purpose of adjusting without displacing the needle. The rear of the magnet is carried sidewise by a screw and moved vertically by a cam, both being operated by a key through a small opening in the side of the case. The spots of light are thus properly adjusted on the slit of the drum box or slideway. For convenience in adjusting, a block on which are two inclined mirrors is placed directly below the opening in the top of the case. In one the galvanometer fronts are in view, and in the other are seen the lines of light on the slits.

When the adjustments are made the galvanometer circuits are closed, and, if films are used, the photograph is taken by pressing the push button seen on the top of the case in Fig. 1. This releases the driving mechanism which brings the drum up to speed and then automatically operates the cylindrical shutter which exposes the film for a single revolution of the drum, beginning and ending the record at the latch that holds the ends of the film. How this is done may be better understood by noting the interior arrangement in the horizontal and vertical projections in Fig. 6, and also Fig. 2, in which it is taken apart showing the inner drum case with its lid removed ready to take out the drum for putting on the film.

A strong spring, wound up by the key shown, actuates a gear wheel which engages a pinion on the drum shaft. Pressing the push button moves a lever and releases the gear wheel for one revolution at the end of which it stops so that the pinion is free to turn in a gap where three or four teeth are cut away and the drum con-

[1] The pole pieces are 2.5 cm. wide and .48 cm. thick, with the ends next to the needle rod tapered down to about 3 mm. wide and 1 mm. thick. These tapered points are not shown in Fig. 6, owing to the small scale.

tinue the high speed attained. Just as the gear stops it releases an arc-shaped arm from the cylindrical shutter which then makes a rolling contact with a small roughened wheel on the drum shaft. The parts are so proportioned that the opening in the shutter uncovers the slit during one revolution of the drum; the arc subtended by the opening in the cylinder being the same as that over which the arm is carried by one revolution of the roughened wheel.

If the slideway shown in Fig. 3 is used instead of the drum, it is operated by giving the thumb nut at the end of the spring a one-fourth turn either way, thus releasing the spring which withdraws a pin on which the plateholder rests. The plate is then shot downward by a strong spring acting on the pin in the slot at the top. The spring ceases to act upon the plate just before it comes to the slit. A vent at the bottom for the escape of the air can be adjusted so as to maintain the speed of the plate very nearly uniform while passing the slit; probably within one-tenth of one per cent. A latch keeps the plate from bounding back after striking the felt pad at the bottom.

Since the rays from the two needles pass through the slit at an angle it is necessary to have the plate pass very close to the slit to get the true phase relations of the two curves. To do this the plate holder is made as shown at the top of Fig. 2. It takes an eight-inch plate of any width up to five inches, and brings it within .8 mm. of the slit.

In Figs. 7–15 are shown a few curves representing current and electro-motive force from alternators having different wave forms, and different kinds of external circuits containing resistance, self-induction or capacity. They were taken on plates 2.5 by 8 inches except Fig. 15 for which a full-sized 5 by 8 plate was used instead of a half plate. The curves are reproduced full size, but only a part from each plate is shown.

Figs. 7 and 8 were obtained for the purpose of testing the accuracy with which the phase relations of E and I are represented. The connections to one galvanometer were reversed so as to reverse the deflection and get a sharply defined crossing of the lines instead of superposition to indicate no lag when incandescent lamps are used. The intersections are found to lie in the same straight

Fig. 1.

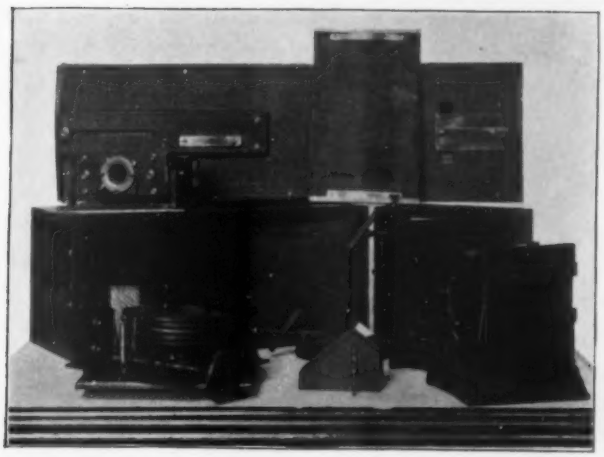

Fig. 2.

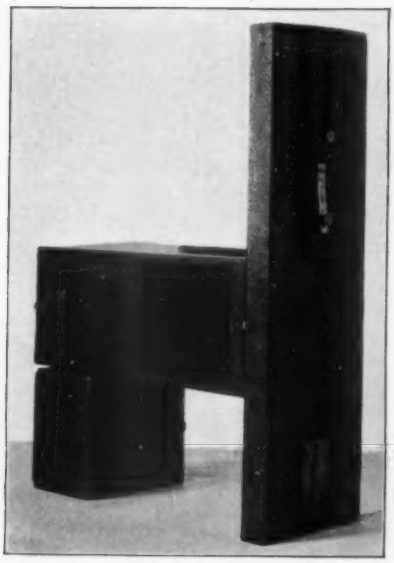

Fig. 3.

Fig. 4.

HOTCHKISS: Alternating Current Curves.

UNIV. OF MICH.

line. This indicates that the phase relations shown by the instrument are reliable.

At first the voltmeter was placed below the ammeter, thus displacing the E curve relative to the other, about .2 mm. in the direction of lag. This error was further increased slightly by the self-induction of the fine wire coils of the voltmeter. Fig. 9 was taken with the voltmeter below. The slight lag of the E curve is probably due to these errors. By placing the voltmeter above the ammeter, as used later for Figs. 7 and 8, these errors tend to compensate and all of the tests made indicate that the compensation is quite exact.

The curves in Fig. 8 have their lines of symmetry slightly separated, due to an accidental displacement of the zero of one galvanometer just before the exposure. The self induction in the armature of the alternator makes the notches in the current curve of less depth than those in the other. The slightest perceptible variations of the form or shading of any one wave from that of the others of a single revolution have been found to be exactly reproduced in the corresponding wave of the next revolution.

The laboratory transformers, used for Figs. 7, 9 and 13 are operated from a 2200-volt General Electric Co. alternator running at a frequency between 128 and 130 cycles per second. The following tables gives for the various generators used some approximate values for frequency and virtual current and electro-motive force.

TABLE I.

Fig.	Generator.	I	E	Frequency.
7	General Electric Co. Alt. and Tr	——	55	130
8	Westinghouse Pony Alt.	——	——	135
9	General Electric Co. Alt. and Tr.	10.4	96	130
10	Brush Alt. and Tr.	0.33[3]	110	115
11	Special Alt.[1]	13.2	27	65
12	" "	12.5	29	60
13	General Electric Co. Alt. and Tr.	10.0	106	130
14	Sprague Alt.[2]	——	17	17
15	" "	——	30	17

[1] Designed and constructed by students in Sibley College, Cornell University.

[2] A 10-H. P. Sprague D. C. motor changed to a 3-phase generator by putting three rings on the commutator. A single phase was used.

[3] Computed.

The capacity for Fig. 10[1] was about 4.2 microfarads, being three Stanley condensers in parallel. The resistance in the voltmeter circuit was about 120 ohms.

In Fig. 12 the ripples on the *E* curve, due to the grooved armature core, are entirely removed from the current curve by the solenoid in the circuit. In Fig. 13 also the forms of the curves differ very much due to the solenoid.

The electrodes of the electrolytic cell used for Figs. 14 and 15 consisted of an arc light carbon with a helix of No. 10 aluminium wire around it and about 1 cm. away. These were placed in a saturated solution of alum with a little sulphuric acid added. The zero line was traced by a fine wire placed across the slit in the slideway. The shadow of the wire made a break in the curve every time the beam of light crossed it.

The cell allows a comparatively large current to flow from carbon to aluminium, but offers great opposition to the flow of current from aluminium to carbon in the cell. The *E* curve shows the difference of potential around both the cell and the current galvanometer; hence the fall of potential in the latter must be subtracted from the ordinates of the *E* curve to give the difference of potential impressed at the terminals of the cell. One centimeter deflection represents .555 ampère for the *I* curve, and 10.82 volts for the *E* curve. The fall of potential in the current galvanometer is 9.35 volts for each centimeter of ordinate of the current curve. Fine wire coils of high resistance were used for both galvanometers for Figs. 10, 14 and 15.

From the data given the values in the following table were obtained for the parts of the curves above and below the zero lines in Figs. 14 and 15.

The lack of symmetry of the *E* curve is due to the drop in the resistance (about 1 ohm) between the generator and the apparatus.

The curves may be made heavy, or light and narrow. To obtain narrow lines with sunlight it is directed upon the needle mirrors from reflectors, about 1 mm. wide, placed about 1 meter from the

[1] For correct phase relations turn the right end to the left, then imagine one curve rotated 180 degrees about its axis of symmetry. One pair of galvanometer terminals were reversed by mistake, and no later opportunity was found to get a correct one as the Brush generator was used for only a short time during some repairs.

needles. The slit in front of the plate is about .5 mm. wide. The line of light across the slit may be made even narrower than that. No lenses are used. The small mirror gives an image in much the same way as a pinhole camera.

TABLE II.

	Fig. 14.		Fig. 15.	
	Above.	Below.	Above.	Below.
Maximum volts for *E* curve	24.	23.	42.5	40.7
" ampères for *I* curve	.07	.98	.39	1.76
" volts on cell	23.5	6.6	36.	11.
Volts for zero current	+23.	−4.	+30.	−4.
Direction of current in cell	Al. to Carb.	Carb. to Al.	Al. to C.	C. to Al.

When using arc lights no reflectors need be used, but narrow slits should be placed close to the arcs. One arc lamp and one reflector may be used instead of two arcs. The pair of hand-fed arcs shown in Fig. 4, have been found to be very convenient, being light and easily adjusted. Turning the screw changes the length of the arc. Pulling and pushing the screw moves the arc up and down on the slides. The bottom of the metal slit is bent around the lower carbon. The maximum intensity of light is nearly horizontal on the side towards the slit. The round windows in the upright are covered with colored glass for viewing the arc while adjusting it. The upright is covered with asbestos board. The terminals on the back are connected to the metal slides. The screws are insulated from the circuits.

That the deflection is very nearly proportional to current is evident from the calibration curves in Fig. 16. The ordinates of the ammeter curve are plotted from the top downward so as to separate the curves. The slight excess of deflection for large currents can probably be easily eliminated by changing the form of the soft-iron pole tips. An earlier form with longer air gap and an electro-magnet gave exact proportionality.

The instrument shown in Figs. 1–3 is the only one that has been made.[1] It was constructed (except the needles and slideway) by Mr. F. C. Fowler, the instrument maker to the Department of

[1] They are to be manufactured and sold by Elmer G. Willyoung & Co., Philadelphia, Pa.

Physics. The ingeniously arranged driving mechanism and shutter for the drum-box were devised by him to replace the one designed by the writer and is a great improvement.

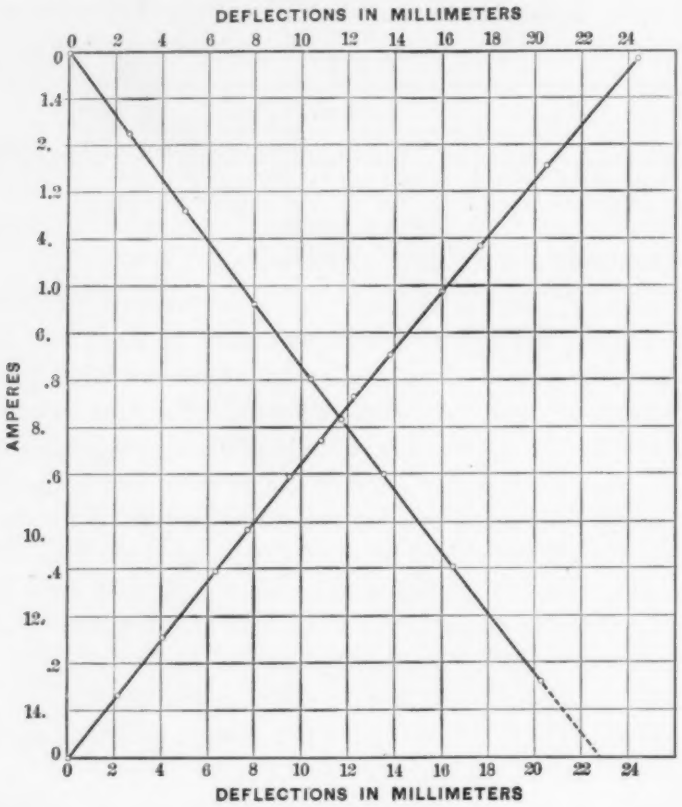

Fig. 16.

The original design included a vibrator of known frequency with a small mirror attached for tracing either a curve on the plate as a time record, or a straight datum line with the mirror at rest. Another part, not yet completed, is an automatic switch for opening or closing the circuit or short-circuiting any part of it while the plate is moving past the slit. It is intended also to have an electromagnet that will be interchangeable with the permanent magnets for experiments in which a very strong field is desired. Apparatus of different forms for special uses has been devised and is in progress.

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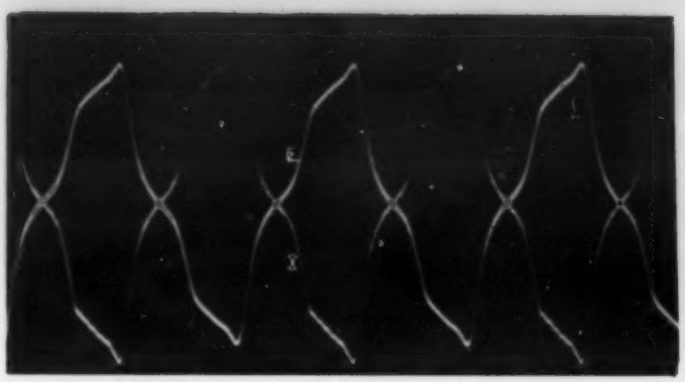


FIG. 7. GENERAL ELECTRIC CO. ALTERNATOR. - INCANDESCENT LAMPS ON TR.
ONE GALVANOMETER REVERSED TO SEPARATE CURVES.

FIG. 8. WESTINGHOUSE PONY ALTERNATOR. - INCANDESCENT LAMPS.
ONE GALVANOMETER REVERSED.

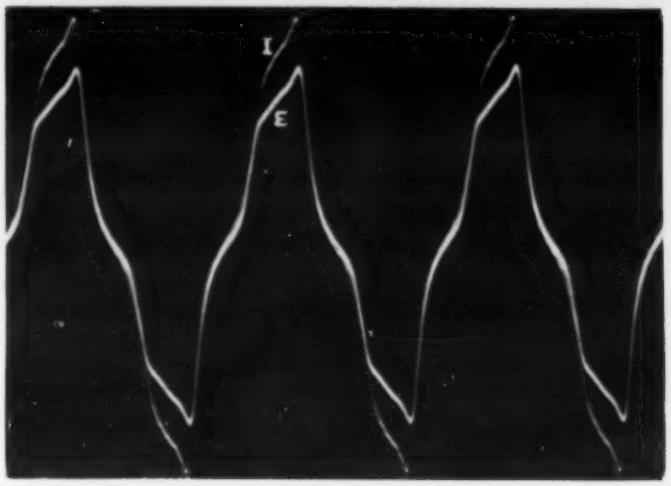


FIG. 9. GENERAL ELECTRIC CO. ALTERNATOR. NON-INDUCTIVE RESISTANCE ON TR.

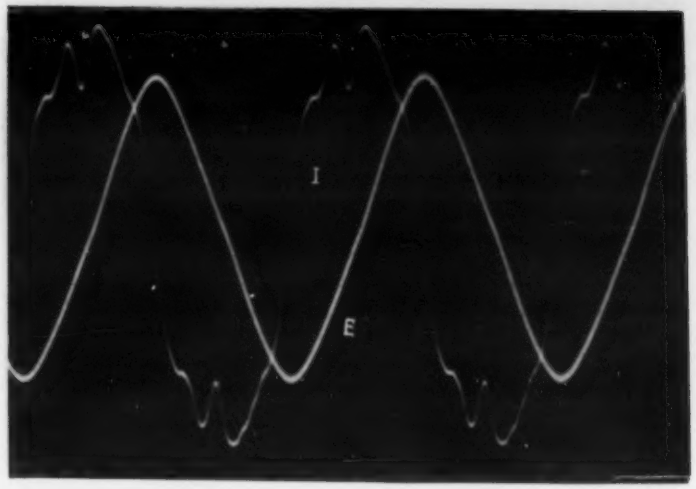


FIG. 10. BRUSH ALTERNATOR. CONDENSER ON TRANSFORMER.

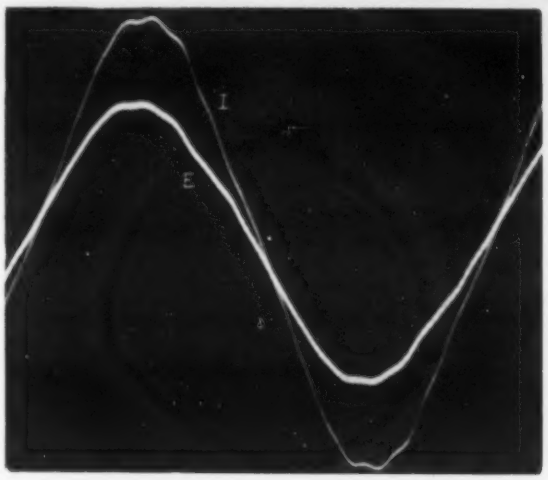


FIG 11. SPECIAL ALTERNATOR. - NEARLY NON-INDUCTIVE RESISTANCE.

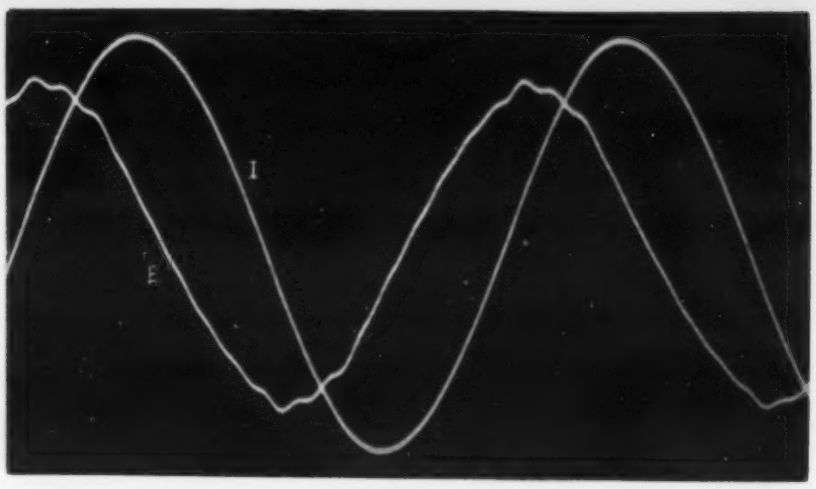


FIG. 12. SPECIAL ALTERNATOR, - SOLENOID. NO IRON.

To face page 160.

FIG. 12. SPECIAL ALTERNATOR, - SOLENOID. NO IRON.

FIG 11. SPECIAL ALTERNATOR. - NEARLY NON-INDUCTIVE RESISTANCE.

FIG. 14. CURRENT DUE TO AN ALTERNATING E. M. F. IMPRESSED ON A CELL WITH CARBON-ALUMINIUM-ELECTRODES.

FIG. 13. GENERAL ELECTRIC CO. ALTERNATOR. SOLENOID ON TR.

FIG. 15. SAME AS FIGURE 14, EXCEPT HIGHER E. M. F. 38 VOLTS MAXIMUM ON THE CELL.

H. J. HOTCHKIS,

APPARATUS FOR PHOTOGRAPHING ALTERNATING CURRENT CURVES.

UNIV. OF MICH.

ON CONVECTION CURRENTS AND ON THE FALL OF POTENTIAL AT THE ELECTRODES IN CONDUCTION PRODUCED BY RÖNTGEN RAYS.

By John Zeleny.

I. Convection Currents Produced by Röntgen Rays.

THE passage of electricity through a gas under the influence of Röntgen rays is accompanied by movements of the gas itself, which are of considerable magnitude.

The presence of these convection currents can readily be made visible by means of particles suspended in the gas, and for this purpose it has been found convenient to use the ammonium chloride particles formed by the action of ammonia and hydrochloric acid.

One form of apparatus used for carrying on the experiments is represented in elevation in Fig. 1.

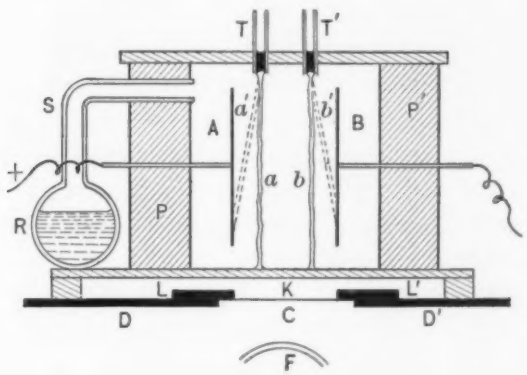


Fig. 1.

The two brass plates A and B are each 6.3 centimeters high and 2.8 centimeters wide and act as electrodes in the gas, being connected to the opposite poles of a battery of storage cells. The plates are enclosed in a box of which the sides P and P' are formed of paraffin blocks, while the other two sides are glass plates, which leave the whole interior open to observation. The bottom K of the

box is made of wood to permit the entrance of the Röntgen rays, which pass from the source F through the aluminium window C in the cover DD' of a lead-covered box which contains the induction coil and tube. The part of the space, in the box PP', that is exposed to the rays can be regulated as desired by moving the lead strips L and L', while observing the fluorescence on a screen placed above the apparatus.

The glass bulb R contains aqua ammonia from which the tube S conducts the ammonia gas into the box PP'. Fastened to the cover of the latter are the two glass tubes T and T', into which drops of hydrochloric acid are introduced. The particles of ammonium chloride formed at the lower ends of the tubes, where the acid is in contact with the ammonia in the air, descend in the box with slow velocity, producing well-defined whitish streams a and b near the plates A and B.

While the air in the box is at rest the particles move vertically downwards, and this motion is maintained when the plates are kept at the same potential and the air between them is exposed to Röntgen rays, or when the plates are at different potentials and the air is not exposed to the rays.

If, however, while the plates are maintained at different potentials, the whole space between them is exposed to the rays the two streams are deflected or bodily carried each to its neighboring plate.

When the rays are turned on, the streams of particles begin to move from their initial position with a velocity that is accelerated for an appreciable length of time, *i. e.*, the particles do not assume a uniform motion at all quickly.

For a given intensity of the rays, the rate of the motion is dependent upon the potential gradient between the plates.

When the electric field is not too intense the streams assume a steady state in oblique positions as indicated by a' and b' in Fig. 1. In this case the horizontal component of the motion is not large compared to the vertical one. But if the gradient is made much larger, the horizontal velocity may predominate greatly and the streams are carried bodily to the plates. The potential differences used in these experiments ranged from 40 to 320 volts for a distance of two centimeters.

It becomes necessary to show that the movements indicated by the streams are true indices of the actual motion of the gas, that the gas is set into motion and carries the particles with it, and that these are not themselves the primary cause of the motion.

If the ammonium chloride particles became charged in some way we might expect them to partake of movements somewhat similar to those observed.

But such charges are not acquired during formation, for in the absence of the rays the motion of the particles is not influenced by the electric fluid, and particles are equally affected when the rays are turned on, whether they were formed before or after the rays are allowed to act.

Any charges acquired from the ions produced in the gas do not give rise to the observed motions, for if both kinds of charge are present, the streams should separate into two parts, one going to each plate; and if on the other hand, near each plate the particles get a charge opposite to that of the plate, we could not thereby explain an experiment to be described later, where both of the streams are drawn to the same plate.

While it thus seems clear that the motion of the streams is a fair representation of what is going on in the gas when the particles are not present, still in order to test this point further the following experiment was performed.

A carbonic acid gas generator was connected to a glass tube which ended in a small opening and projected upwards through the bottom K of the box PP' in Fig. 1. A narrow jet of carbonic acid gas could thus be sent upwards in the neighborhood of and parallel to one of the plates.

The jet was made visible due to its different refractive index, by projecting the whole apparatus with an arc-light as a shadow upon a screen.

With some care in choosing the proper orifice of efflux it was possible to obtain a portion of the jet, which maintained a distinct form even though its upward velocity was not large compared to the motions under investigation.

When the rays were turned on, while the plates were at a different potential, the jet of carbonic acid gas was deflected towards its neigh-

boring plate, the same effect as was obtained by using the ammonium chloride particles. Since, however, a gas molecule cannot become electrified the result here obtained cannot be ascribed to any secondary effect, and it is evident that during conduction the gas itself is set into motion.

It is well known that gas movements accompany the passage of electricity through gases in some other cases, and they are explained as due to the movement of electrified carriers through the gas.

In seeking to find the cause of the convection currents here under consideration we are at once led to ascribe them to the motion of the charged ions by means of which the conduction is carried on.

When an ion starts moving through a gas under the action of an electromotive force, its resulting velocity is assumed to become constant in a very short time. All of the work now done by the field upon the moving ion is transferred to the body of the resisting gas, which therefore tends to move with the ion.

If an equal number of positive and negative ions are moving in opposite directions through a gas the resulting velocity is nil, but if the ions of one sign are more numerous, that is, if a free charge exists in the gas it then tends to move with accelerated motion in the direction taken by the predominant ions.

Now in the case of conduction under the influence of Röntgen rays it has been shown by J. J. Thomson and E. Rutherford[1] that a free charge exists in the gas, and the writer[2] has investigated the distribution of the charge between two plates, and found that near each plate there is a charge in the gas opposite in sign to that of the plate, and that this decreases rapidly with the distance from the plates.

In the neighborhood of each plate, therefore, there exists a force tending to move the gas towards the plate, while near the center there is a neutral region.

If the two plates were unlimited in extent, then the motion of the gas would produce an inequality in the gas pressure which would counteract all further movements ; but with limited plates the excess pressure at their surfaces is relieved sideways, and there results a set of convection currents whose velocity increases until the loss of

[1] J. J. Thomson and E. Rutherford, Phil. Mag., Nov., 1896, p. 394.

[2] J. Zeleny, Phil. Mag., July, 1898, p. 138.

energy due to friction is equal to that gained from the moving ions. In accordance with this view, in the experiments described above it was seen that when the conduction begins the gas starts to move towards the electrodes, that this motion is accelerated for a noticeable length of time, that it is more vigorous with greater ionization and greater potential gradients, and that finally it assumes a steady state.

The configuration of the convection currents in the apparatus during the steady state has been traced in some cases, but is of no especial interest as it is so dependent upon the form and size of the apparatus used.

The view taken of the nature of the convection currents has been tested by a considerable number of experiments made with different forms of apparatus and under varying conditions; it may be useful to mention here a simple modification of the experiments already described:

If in the apparatus shown in Fig. 1, the rays instead of falling upon all of the gas between the two plates are allowed to impinge only on a part next to one plate, as may be done by screening the other side by one of the movable strips of lead L, then as soon as the rays are turned on both streams of ammonium chloride particles move towards the plate from which the rays are screened. By cutting off the rays from one side of the field the gas in the exposed part, which normally moves towards its adjacent plate, now has its motion reversed so that all the gas between the plates is moving towards the screened plate.

If the distribution of the ions in this case is considered, it is seen that this is a necessary consequence. For, of the ions produced in the space exposed to the rays, those which under the electric field move towards the screened side, have a longer distance to travel, and while they are in the screened space they constitute a much larger free charge than is normal, for here there are not present any of the ions of the opposite sign which usually neutralize the greater part of the effect.

The motion of this abnormally large charge produces an air current which sets into motion all of the air between the plates, and even overcomes the force near the exposed plate which tends to produce motion in the opposite direction.

It has been stated that the velocity of the convection currents is accelerated for an appreciable time, so that by finding this acceleration it is possible to get a rough idea of the magnitude of the free charge necessary to produce the observed movements. Thus in a particular case where the field was uniformly exposed to the rays and where the potential gradient was about 60 volts per centimeter, the stream of particles was observed to move 5 millimeters from rest in two seconds. Assuming the acceleration uniform for this short period of time this gives its value as .25 cm. per sec. per sec. Assuming further that the charge in each cubic centimeter has only to move this volume of air, although it actually has to move rather more, and calling δ the average density of electrification we have, approximately, by equating the two expressions for the force acting

$$\delta \times \frac{60}{300} = .25 \times 1.2 \times 10^{-3}.$$

This gives for δ a value of about 10^{-3} a number of the same order as has been observed for the free charges existing in these cases.

II. THE POTENTIAL GRADIENT AT THE ELECTRODES.

It has recently been shown independently by the writer[1] and by Child,[2] that when conduction is going on between two plates the potential gradient between them is not uniform and that at the plates there is a rapid fall of potential.

In these investigations, however, no experimental points were obtained on those parts of the gradient curves which correspond to these rapid changes at the plates, so that it is not possible to say whether the fall exists in the gas near to the electrodes or whether it is due to the formation of some kind of a double layer on the plates themselves.

If in passing from the gas to the electrode the potential changes in a continuous manner then evidently its variations can be explained by the presence of free charges in the gas, but if on the other hand there is a sudden change of potential at the electrode it is necessary

[1] J. Zeleny, Phil. Mag., July, 1898, p. 147.

[2] C. D. Child, Wied. Ann., 65, p. 152, 1898. Also PHYSICAL REVIEW, 6, p. 285, 1898.

to assume the presence, at the surface, of a charged layer opposite in sign to that of the metal itself.

In the previous work the nearest point at which the potential was determined by the writer was one millimeter from the plate, while the nearest observed by Child was five millimeters distant.

And since it has been shown[1] that the free charges in the gas increase rapidly as the electrode is approached, it was thought possible that a determination of the potential at points nearer to the surface than had been done might throw some light upon the problem just mentioned.

For this purpose a form of apparatus was constructed which possesses some important improvements and permits of a determination of the potential at points within $\frac{1}{10}$ millimeter from the electrode.

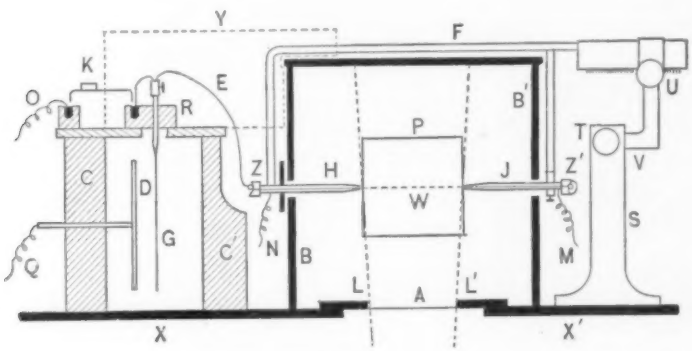


Fig. 2.

The apparatus is shown in elevation in Fig. 2, as it was set up on the cover, XX', of a lead box which contained the induction coil and Crookes' tube.

The electrodes between which the potential gradient was investigated were made of brass plates, 7.3 centimeters square, which had been carefully worked flat.

One of these is represented by P, the other one being parallel to it, and both being supported by rods (not shown) which were fixed at right angles to them.

The plates were kept at a constant difference of potential by a

[1] J. Zeleny, Phil. Mag. July 1898, p. 138.

battery of cells, and were completely surrounded by a lead box BB'. The potential of points between the plates was investigated, during conduction, by means of a fine brass wire W, $\frac{1}{10}$ of a millimeter in diameter, which was supported by a frame F. This frame was fastened to a microscope stand S, which allowed, by means of the set screws T and U, of two motions at right angles to each other. The position of the wire was determined by means of a vernier at V.

The parts of wire not included between the plates were surrounded by the brass guard tubes H and J, which tapered to a small size at their ends.

The wire was stretched taut between two small insulating rods which were carried by the parts Z and Z'.

It was connected by means of the wire E with the gold-leaf G, which, supported by the paraffin block R, hung in a box of which C and C' formed two sides, the other two being of glass.

Parallel to the gold-leaf was the adjustable brass plate D, which was connected to earth through a high resistance by the wire Q.

The lead cover Y served as a screen for a part of the apparatus. Earth-connected metal screens also covered the exposed sides, but are not shown in the figure, nor is the microscope, by means of which the movements of the gold-leaf were observed, and which carried in the focus of its eye-piece a finely divided scale.

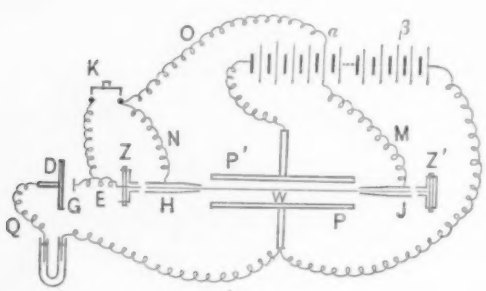


Fig. 3.

The electrical connections can be seen better from Fig. 3 where the apparatus is viewed from above and where the letters have the same significance as in Fig. 2.

By pressing the key K, the wire W and the gold-leaf G, can be simultaneously charged to any required potential by means of the wire O leading to a battery of cells β. The guard tubes H and J are also kept at the same potential by the wires N and M, and the same battery maintains the plates P and P' at a constant difference of potential.

The gold-leaf method here employed for detecting the changes of potential of the wire, although in the form used it does not detect as small differences as a quadrant electrometer, still has an advantage in that, owing to the small capacity of the system, it takes up the desired potential of a place much more rapidly, which is a great convenience when working with the small conductivities here concerned.

By making the distance between the plate D and the gold-leaf G, such that for the voltage to be used the leaf is almost on the point of being drawn to the plate, the indications are sufficiently sensitive for the purpose in hand.

The following is the procedure for finding the potential of any place between the plates during conduction.

The investigating wire W being placed in the desired position, the wires O and M (Fig. 3) are connected to the battery β to a point α which has a potential estimated to be that of the place in question.

The key K is now opened, leaving the wire W and the leaf G insulated and charged to a known potential. The rays are now allowed to pass through the aluminium window A and to traverse the space between the plates, and according as the wire was charged to a lower or higher potential than that of the required point the gold-leaf shows an increase or decrease in deflection, the wire gaining a charge from or losing it to the gas.

The wires O and M are connected successively to different points of the battery until for two adjacent cells the gold-leaf indicates an increase in charge for one and a decrease for the other, when the rays are turned on.

By noting for the steady state in these two cases, the number of divisions of displacement on the microscope scale, the desired potential is obtained by interpolation.

The presence of the guard-tubes H and J was found to be important, since being kept at the same potential as the insulated wire W, it is impossible for this wire to gain or lose a charge by gas conduction at any point except that between the plates whose potential is desired.

Without the guard-tube a small leakage is liable to occur from those parts of the wire which are outside of the plates, although

the direct rays do not impinge upon any of the lines of force from these parts. Ions reach these places, however, either by the convection currents described in §I or by diffusion or by secondary radiation.

In the results obtained with this apparatus, the fall of potential at or near the plates was found to be less than that hitherto observed, and especially no approach could be made to such high values as those described by Child (loc. cit.).

There is good evidence of the lack of accidental leakage, for with the same potential difference between the plates the nature of the potential curve was found to be independent of the absolute potential of the plates.

The following table gives the results obtained for the potentials of some points near to the zero plate, when the plates were 1.215 centimeters apart and were kept at a difference of potential of 320 volts.

TABLE I.

Distance from zero plate.	Potential observed.	Potential calculated for uniform gradient.	Difference.
.00 cm.	+ 0 volts.	+ 0 volts.	0 volts.
.01	8.7 "	2.6 "	6.1 "
.02	11.9 "	5.3 "	6 6 "
.03	14.8 "	7.9 "	6.9 "
.04	17.4 "	10.5 "	6.9 "

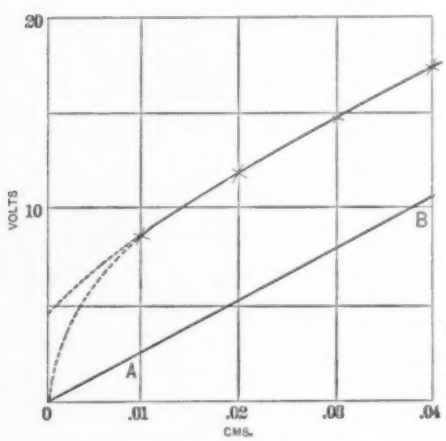


Fig. 4.

These results are shown graphically in Fig. 4, when the line *AB* represents the uniform gradient existing when the rays are not acting.

For the same distance between the plates and smaller voltages which were tried down to 40 volts, the fall in the first $\frac{1}{10}$ mm. was found to be correspondingly smaller.

It is seen that in the first $\frac{1}{10}$ mm. from the plate the change of potential is about as large as in the following $\frac{3}{10}$ mm., and it thus appears that the fall of potential near the electrodes is abnormally large.

The results do not permit us to say, however, whether this is due to a charge adhering as it were to the metal surface or whether it is due to the existence of a charge in the gas within the first $\frac{1}{10}$ mm. from the electrode.

If it were possible to measure the electric force acting upon the plates during conduction such a distinction could be made by comparing the potential gradient thus found at the plate with that obtained in the above experiments for points near to it. The presence, however, of the convection currents described in § I, interferes with such a measurement.

Various attempts were made to get rid of the effects of their pressure, but no reliable method was obtained.

Still, it can be shown that during conduction there s an increase in the force acting upon the electrodes.

This is most readily done with a modification of the apparatus shown in Fig. 1, which is represented in Fig. 5, where the box PP' which encloses the plates A and B is not drawn.

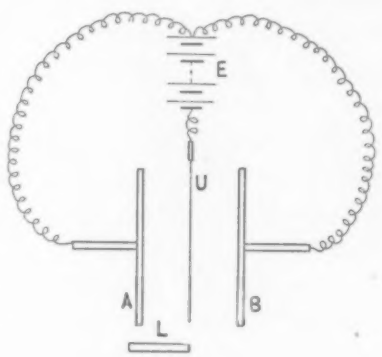


Fig. 5.

U is a gold-leaf suspended midway between the plates and is connected to a pole of the battery E to the other pole of which the two plates are joined.

The motions of the gold-leaf are observed by means of a microscope.

When everything is arranged systematically and the rays have access to the whole space between A and B there is little or no motion of the leaf when the rays are turned on.

If by means of the lead plate L the rays are cut off from the space between the leaf and one of the plates, say A, then the gold leaf is deflected towards the side traversed by the rays, that is towards B.

The attraction is therefore greater on the side where conduction is going on than where it is not.

When the plate B is removed altogether, the leaf is deflected towards A by the existing electrostatic attraction.

Allowing the rays to pass between the plate and the leaf increases this deflection so that in a particular case with a distance of 1.5 centimeters between the two, the passage of the rays increased the deflection by an amount equal to that produced by an addition of ten volts to the existing potential difference of eighty volts.

This does not indicate the total increase in the potential gradient at the plate, because the leaf is also acted upon by a counteracting pressure due to the gas motion described in § I, but as it is difficult to estimate the value of this pressure, all that the above experiment enables us to say is that the gradient at the leaf when the rays were acting was *more than* $\frac{9}{8}$ of the gradient existing previously.

It is possible to arrange the above apparatus so that an air current will assist the deflection of the gold-leaf. When both plates are used, if instead of being kept at the same potential, one, say B, is connected to the leaf, then if the rays are passed between the leaf and this plate, the leaf is deflected towards the plate, although this is at the same potential.

Two effects combine to produce this result. Because of the small width of the leaf some lines of force near the sides of the box extend from A to B, and others starting from A reach the leaf U on the side turned towards B.

As these latter are in the path of the rays, there are some ions reaching the leaf from and producing a deflection towards B.

It was also found that the convection currents in the gas at the sides of the box were from B to A, but in the center between A and U there was a return current directed from A towards U, which on reaching the leaf, went around both edges of it, thus tending to move the leaf towards B.

The results of the investigations described in this paper may be summarized as follows:

I. During conduction through a gas under the influence of Röntgen rays, convection currents are produced in the gas, which, in general, moves towards the electrodes.

II. The convection currents are caused by the motion of the free charges existing in the gas.

III. The rapid fall of potential near to the electrodes during the conduction exists within the first $\frac{1}{10}$ mm. from the surface.

IV. The electric force acting upon the electrodes is increased by exposing the gas to the Röntgen rays.

I desire, in conclusion, to express my thanks to Professor J. J. Thomson for many valuable suggestions during the course of the investigation.

CAVENDISH LABORATORY, October 31, 1898.

A MECHANICAL MODEL OF THE CARNOT ENGINE.[1]

BY JOHN C. SHEDD.

THERE is, perhaps, no proposition in the range of Physics that is more difficult of comprehension by the average student than that embodied in the so-called *Carnot Engine* and *Carnot Cycle*.

The reason for this is largely due to the difficulty of illustrating by experiment an engine that is not supposed to work under *practical* conditions, together with the too frequent conclusion of the student, that the matter is, therefore, devoid of *practical* importance.

Since the Carnot engine is an ideal engine, any apparatus intended to illustrate it must be of the nature of a *model*, and not of a "*working machine*." With this limitation, however, the following form of apparatus has been found successful in illustrating the following points:

I. The successive operations of the Carnot cycle.

II. The reversibility of the Carnot engine.

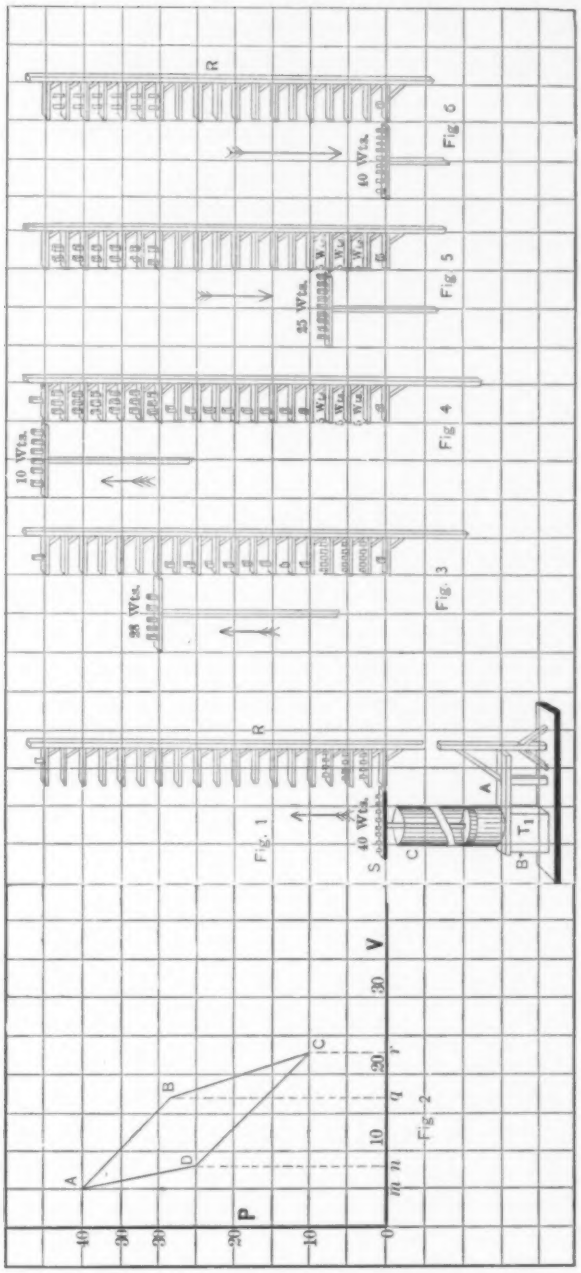
III. That a quantity of heat is transferred from the hot to the cold body (or visa versa), and that work is done by (or upon) the working substance, in passing through the complete cycle.

IV. That the adiabatic operations are equal and opposite, and that therefore the *work* is accomplished during the isothermal expansion (or contraction, depending upon direction in which the engine is working).

V. That the efficiency of the Carnot engine depends (1) upon the difference of temperature between the hot and cold bodies, and (2) upon the absolute temperature of the hot body.

The apparatus is shown in Fig. 1. Cylinder C is supported upon a stand, A, such that the cube B can be placed under it. The cube B is intended to represent the heat source at temperature T_1, the refrigerator at temperature T_2, or the non-conducting substance according to the side presented. The piston P is operated by the work-

[1] The feasibility of such a model was first suggested to me by Mr. R. W. Wood.


R
Fig. 6
40 Wts.
Fig. 5
25 Wts.
Fig. 4
10 Wts.
Fig. 3
28 Wts.
Fig. 1
40 Wts.
S
C
A
B
T1
V
P
A
B
C
D
Fig. 2

ing substance, and supports the shelf S. As P rises S comes successively opposite the shelves on the rack R.

The rack R consists of a convenient number of shelves (*e. g.*, 19), upon which weights are disposed in a predetermined manner (*e. g.*, as shown in Fig. 1). On the shelf S are placed a number of weights also predetermined (in the present case 40) and the cycle is supposed to begin with S opposite the lowest shelf on R, as shown in Fig. 1.

I. Referring to Fig. 2 we shall see how the model follows the Carnot cycle.

Commencing at the point A on the diagram, om represents the volume of the cylinder below the piston and mA the pressure due to the 40 weights on the shelf S, the piston and shelf being supposed without weight.

We will let $T_1 = 402°$ and $T_2 = 270°$ both on the absolute scale.

First Operation.—Place the block B so that it shall present the side T_1, thus signifying that the cylinder is in communication with the source of heat. Remove *one* weight from shelf S to the shelf opposite on R; since the weight has been displaced horizontally no work has been done, and since the pressure has been decreased on the piston, it rises until equilibrium of internal and external pressure is restored. Let this be when S is opposite the second shelf on R, and let *twelve* weights be successively removed from S in this manner, thus tracing the isothermal line AB; the position of S being now shown in Fig. 3.

Second Operation.—Turn the block B so as to present the "non-conducting" side, and again allow the piston to rise by removing weights. We first observe, however, that since no heat is received, the working substance will be cooled by further expansion, and, therefore, the pressure must, for a given increase of volume, be decreased faster than in the first operation. Let this be done by removing *three* weights at each step, until *six* spaces have been passed and *eighteen* weights removed.

During this operation the adiabatic line BC has been traced and the temperature of the working substance has fallen from T_1 to T_2. The position of the shelf S is shown in Fig. 4.

Third Operation.—Turn the block B so as to present the side T_2, and

force the piston down by successively adding weights to the shelf S from rack R. Since the cylinder is in communication with the refrigerator the contraction is isothermal, and there is a flow of heat from the working substance to the body B. Since heat is given up by the working substance as fast as developed, the contraction will be greater for a given change of pressure, than in the *second operation*. This may be represented by adding but *one* weight, as the piston passes each shelf. Let this be done until *fifteen* weights have been replaced upon S and *fifteen* spaces have been passed over. During this process the isothermal line CD is traced and the shelf S is brought to the position shown in Fig. 5.

Fourth Operation.—The adiabatic contraction DA is accomplished by adding *five* weights at a time until *fifteen* weights have been added, and the piston is returned to its starting point, with the original number of weights upon S.

The working substance is thus restored to its original volume and pressure, and the cycle is complete. The final position of S is shown in Fig. 6.

II. To show the reversibility of the Carnot engine the weights on R should be disposed as in Fig. 6, and the preceding operations gone through with in reverse order.

III. During the *first operation* above, a flow of heat takes place from the hot body to the working substance. Since the temperature is not raised this heat is consumed in doing mechanical work. This work is represented on the diagram by the area $ABqm$, and on the model by the weights raised against the force of gravity. This work is equal to $\frac{39 + 28}{2} \times 12 = 402$ units of mechanical work. The amount of heat drawn from the hot body is then $402\ J$, when J is the mechanical equivalent of heat.

During the *third operation* a counter flow of heat is set up, from the working substance to the cold body. This heat is developed by the mechanical work done in forcing down the piston, and is represented by the area $CpnD$ on the diagram, and on the model by $\frac{11 + 25}{2} \times 15 = 270$ units of mechanical work. This mechanical work is all consumed in heat and the amount of heat that flows into the cold body is $270\ J$.

We thus see that $270\,J$ units of heat have been transferred from the hot body to the cold body, while $(402 - 270)\,J = 132\,J$ units of heat have been left in the form of mechanical work.

IV. During the *second operation*, the working substance is transforming part of its heat into mechanical work by raising the piston. The amount of this work, as measured on the model, is $\frac{25+10}{2} \times 6 = 105$ and is represented on the diagram by the area $BCpq$. The heat transformed is then $105\,J$. During the *fourth operation* work is done upon the working substance by the descending piston in amount equal to $\frac{30+40}{2} \times 3 = 105$. The heat added to the working substance is therefore $105\,J$, which is exactly what it lost during the second operation.

If we now sum up the positive and negative work done during the cycle, we get on the diagram the area $ABCD$ and on the model

$$402 + 105 - 270 - 105 = 132 \text{ units of mechanical work.}$$

This is even more clearly shown directly by the model; for the eye immediately observes the difference between the initial position of the weights in Fig. 1, and their final position in Fig. 6.

We see that the twelve weights, which were at first on shelves Nos. 2, 3 and 4, have been raised to shelves Nos. 13, 14, 15, 16, 17 and 18, while the lone weight on shelf No. 19 has fallen 18 spaces to shelf No. 1. The work accomplished is equal to $14 \times 6 + 11 \times 6 - 18 = 150 - 18 = 132$ units of mechanical work.

V. The expression for the efficiency of the Carnot Engine is

$$\eta = \frac{T_1 - T_2}{T_1}.$$

In what has gone before we let $T_1 = 402,^\circ$ $T_2 = 270^\circ$. The expression then becomes

$$\eta = \frac{402^\circ - 270^\circ}{402^\circ} = \frac{132}{402} = 32.8\%.$$

This is also expressed on the model by

$\frac{\text{Work gained}}{\text{Work taken from heat source}} = \frac{132}{402} = 32.8\%$. We thus see that, by assigning proper values to T_1 and T_2, the value of η developed by the model is the same as that given by the theoretical equation.

The change in the value of η, caused by a change in the value of $T_1 - T_2$ is shown by working out a new cycle. Thus if we let $T_1 = 342°$, $T_2 = 285°$. Let the total number of weights on s equal *thirty-five*, and let the operations be as follows:

1. Remove 12 wts., 1 at a time;
   Work done $= \frac{34+23}{2} \times 12 = +342.$
2. Remove 12 wts., 2 at a time;
   Work done $= \frac{21+11}{2} \times 6 = +96.$
3. Replace 15 wts., 1 at a time;
   Work done $= \frac{12+26}{2} \times 15 = -285.$
4. Replace 9 wts., 3 at a time;
   Work done $= \frac{29+35}{2} \times 3 = -96.$

$\eta = \frac{342° - 285°}{342°} = 16.66\%.$

$\eta = \frac{\text{Work gained}}{\text{Work taken from } T_1} = \frac{57}{342} = 16.66\%.$

Work gained $= 342 + 96 - 287 - 96 = 57.$

We thus see that in changing the range of temperature from 132° to 57°, η has fallen from 32.8% to 16.66%.

The change of η due to change of T_1 and T_2 without changing in the value $T_1 - T_2$ is even more easily illustrated.

Thus in the first cycle, above, let us suppose $T_1 = 582°$ and $T_2 = 450°$. This is accomplished on the model by adding an extra weight of *15* to shelf S, making the total weights *55* instead of *40*.

The addition of this extra weight evidently does affect the initial or final positions of the weights on R, and hence does not change the amount of work gained. It does, however, add to the amount of work done during each part of the cycle, and causes an added amount of heat to be drawn from T_1. The amount of this extra heat is $15 \times 12 = 180$ mechanical units.

The total amount of heat drawn from the heat source is (in mechanical units) $402 + 180 = 582$, and we get

$$\frac{582^\circ - 450^\circ}{582^\circ} = \eta = \frac{132}{582} = 23\%,$$

which shows a loss of 10%. If we let $T_1 = 342^\circ$, and $T_2 = 220^\circ$, by *removing* 5 weights from S, we get $\eta = \frac{132}{342} = 38.6\%$ showing a gain of 5.8%.

The following precautions should be noted in the use of the Model:

1. In order that the two aniabatic operations shall equal each other, it is necessary to properly adjust the cycle. This is most easily done by selecting any convenient set of operations, taking care that the number of steps in the *second operation* shall be greater than in the *fourth.* Then such a weight is added over all as shall make the work in the *second operation* equal to that in the *fourth,* by means of the relation

$$W_2 + XS_2 = W_4 + XS_4 \qquad \text{where}$$

W_2, $W_4 \equiv$ work in 2d and 4th operations without the added weight.
S_2, $S_4 \equiv$ number of steps in 2d and 4th operations, and
$X \equiv$ weight to be added over all.

2. The values of T_1 and T_2 are assigned after working out the cycle making $T_1 =$ No. of units of work taken from hot body and $T_2 =$ No. of units of work given up to cold body. These values will then be in accord with the law

$$\frac{T_1 - T_2}{T_1} = \eta.$$

3. In illustrating the change in η with change of T_1 and T_2, where $T_1 - T_2$ is not changed, the equality between the adiabatic operations is destroyed. This cannot be avoided, but need not impair the value of the illustration.

Finally, as the apparatus is merely a *model,* nothing is claimed for it beyond the scope of *illustration. It, of course, demonstrates nothing.* It is hoped, however, that it may be of help in fixing the *Carnot Engine,* and the *Second law of Thermodynamics,* clearly in the student's mind.

PHYSICAL LABORATORY, UNIVERSITY OF WISCONSIN, January, 1899.

A GENERAL METHOD FOR DETERMINING THE BEST READING OF AN INSTRUMENT FOR LEAST ERROR IN MEASURING A GIVEN QUANTITY.

By Frank G. Baum.

IN making indirect measurements it frequently happens that the probable accuracy of the result varies with the reading of the observed quantity. For instance, in measuring a resistance by comparing with a known resistance on a slide-wire bridge, the error in the result due to a *given* error in reading the bridge wire will be different for different points on the wire. It is desirable, therefore, to derive, if possible, a general method whereby the best reading of the instrument used may be determined.

Let y be the quantity to be measured and x the observed quantity, that is, the reading of the instrument; and suppose that the relation between them is given by the equation

$$y = f(x).$$

Differentiate, and we have

$$dy = f'(x)\,dx.$$

Divide the left-hand member of this equation by y and the right side by its equal $f(x)$, and we get

$$\frac{dy}{y} = \frac{f'(x)\,dx}{f(x)}.$$

Now, neglecting infinitesimals of higher order than dx, dy, the numerator of the fraction is the change in y due to the change dx in x, and the ratio $\frac{dy}{y}$ will, therefore, give the percentage of error in y due to an error dx in the reading of x. In order that the error in the quantity determined by the measurement of x be as small as

possible $\frac{dy}{y}$ should be a minimum. Let

$$z = \frac{dy}{y} = \frac{f'(x)dx}{f(x)}.$$

Derive $\frac{dz}{dx}$ and place equal to zero.

$$\frac{dz}{dx} = \frac{f(x)f''(x)dx - f'(x)dx\,f'(x)}{[f(x)]^2} = 0.$$

(In differentiating, dx is taken as constant.)

This gives the condition

$$f(x)\,f''(x) = [f'(x)]^2,$$

which will determine the best reading of the instrument, if such exists. The method of deriving the result is straightforward; its application to particular cases is extremely simple, as will be evidenced by the examples which follow.

In the slide-wire bridge we have[1]

$$y = f(x) = \frac{Rx}{c - x},$$

$$\frac{dy}{dx} = f'(x) = \frac{cR}{(c - x)^2},$$

$$\frac{d^2y}{dx^2} = f''(x) = \frac{2cR}{(c - x)^3},$$

and our condition is

$$\left(\frac{Rx}{c - x}\right)\left(\frac{2cR}{(c - x)^3}\right) = \left(\frac{cR}{(c - x)^2}\right)^2;$$

cancelling, we get

$$2x = c, \text{ or } x = \frac{c}{2}.$$

That is, the reading should be taken near the center of the bridge wire.

Another good example is the tangent galvanometer; here we have

[1] c = length of bridge wire and x the reading on the end opposite y.

$$y = f(x) = x \tan x,$$

$$\frac{dy}{dx} = f'(x) = x \sec^2 x,$$

$$\frac{d^2y}{dx^2} = f''(x) = 2x \tan x \sec^2 x,$$

and our condition gives us,

$$(x \tan x)(2x \tan x \sec^2 x) = (x \sec^2 x)^2 ; \textit{ i. e.},$$

$$2 \tan^2 x = \sec^2 x,$$

$$= 1 + \tan^2 x,$$

$$\text{or } \tan^2 x = 1, \text{ giving}$$

$$x = 45^\circ.$$

STANFORD UNIVERSITY, CAL.

NOTE ON THE COEFFICIENT OF EXPANSION OF ICE.

BY EDWARD L. NICHOLS.

IN a recent number of THE REVIEW[1] I gave a summary of the best known investigations on the density of ice, and described a redetermination of that constant. The literature of this subject shows that the data concerning the coefficient of expansion of ice are even more vague and unsatisfactory than those relating to its density. Interest in this subject seems to have had its origin chiefly in Petzholdt's first report of measurements which gave a negative coefficient. It was this surprising statement which set Brunner[2] to work in 1845 to determine the coefficient and the density of ice by the method described in my recent paper.

In the same year W. Struve,[3] at the Observatory in Pulkowa, planned measurements upon the linear coefficient for the purpose, as he states in his article, of disproving a result so obviously false, that any Russian peasant who had had an opportunity to observe ice on the rivers in cold weather could have pointed out the error. For this purpose ice was made from air-free water and linear measurements upon cylinders five feet in length were carried out by von Schumacher and Pohrt under Struve's direction. The range of temperatures was between $-1°$ Reaumur and $-22°$ Reaumur and the mean of three observations gave a value for the linear coefficient per degree centigrade of .0000531. Fort,[4] in the same volume of the Annalen in which Struve's paper appeared reported that the first quotations from Petzholdt's experiments were wrong and confirmed Brunner's statements to the effect that the coefficient was positive. No data as to the size were given.

Marchand,[5] likewise in 1845, determined the coefficient by weighing 42 grams of ice in a bottle of mercury which held 1107

[1] PHYSICAL REVIEW, Vol. 8, p. 21.
[2] Brunner, Poggendorff's Annalen, Vol. 64, p. 113.
[3] Struve, Ibid , 66, p. 298.
[4] Fort, Ibid., 66, p. 300.
[5] Marchand, Journal für praktischen Chemie, Vol. 35, p. 254.

grams of that metal. The range of temperature was from 0° to −12°C. These measurements are open to the serious objection that no study of the coefficient of expansion of the glass was made, the value being taken from tables. Plücker and Geissler[1] (1852), to whose work reference has already been made in my paper, determined the coefficients by means of their dilatometer. The linear coefficients as found by these observers are as follows :

Brunner,	.0000375
Struve,	.0000530
Marchand,	.0000350
Plücker and Geissler,	.0000528

The determination of a relative change of this kind would appear to be a simpler matter than the measurement of the absolute density and it seems strange therefore to find so large a discrepancy. The method adopted by Struve, on account of its directness, seems more likely to yield accurate results than that employed by Brunner, in which the value depends upon a knowledge of the coefficient of expansion of glass and of the liquid in which the ice was weighed. In order to carry out Struve's method, however, a climate is necessary which like that of Russia affords very low temperatures in the winter season.

In the month of February, 1899, there were several days at Ithaca during which, while the temperature did not reach the lower range obtained by Struve in his work, namely −27.5° C., the thermometer fell daily to a point which made it seem worth while to attempt to repeat his experiments. For this purpose a bar of ice was cut from a block of artificial ice prepared in the usual commercial manner. This ice block had been made at least four months previously, so that it may be assumed to have reached a fairly stable condition. The bar cut from this piece was about 45 cm. in length, 12 cm. wide and 5 cm. in thickness. It was placed upon the bed of a dividing engine, carrying two microscopes with micrometer eye-pieces. The measurements were made in an unheated room, the temperature of which throughout the entire series of experiments fluctuated between −3° and −17°. In order to secure the proper marks upon which to set the microscopes, two shallow conical holes were sunk 39 cm. apart in the surface of the ice block. In these were de-

[1] Plücker and Geissler, Poggendorff's Annalen, Vol. 86, p. 265.

posited globules of mercury about 1 mm. in diameter. Upon the periphery of these globules it was found that the cross hairs of the eye-piece could be set with much greater accuracy than upon any ruled line on ice or even upon glass or metal. The marks remained in their positions undisturbed for several days, during which time readings were taken about twelve hours apart; as nearly as possible at the time when the temperature reached its maximum and minimum point, *i. e.*, the early morning and in the evening. After several days the wasting away of the surface of the ice block by evaporation disturbed the integrity of the cavities in which these globules were set and they began to wander about the surface. It is probable that if the precaution had been taken to sink a deeper hole, this trouble might have been postponed for a longer time. Five sets of readings were made, the temperature of the ice in each instance being determined by means of a thermometer, the bulb of which was inserted in a horizontal hole so drilled as to bring the mercury approximately at the center of the block. The average value of the coefficient from the five sets, corrected for the expansion of the engine, was $.0000540 \pm .0000020$. The temperature to which this coefficient applies is approximately $-10°$, the mean range between $-8°$ and $-12°$. The range of temperatures in these experiments was not so great as in those described by Struve, nor was the distance between the microscopes nearly so long. The coefficient, however, agrees very well with his as given in the preceding table and also with the value given by Plücker and Geissler. Whether the marked discrepancy between these values and those of Brunner and of Marchand is due to errors arising from the indirect character of the methods employed by the last named observers, or whether there are in reality such notable differences in the coefficients of expansion of different varieties of ice, it will be interesting to determine. The duration of the cold weather necessary to the convenient execution of such experiments did not suffice for the extension of the investigation to other ice blocks and it is at rare intervals only that in our climate weather suitable for such work occurs. It has seemed desirable under these circumstances to report the value obtained, since the continuance of the experiments may be indefinitely delayed.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY, Feb. 23, 1899.

NEW BOOKS.

The Elements of Physics. A Text-Book for High Schools and Academies. By ALFRED PAYSON GAGE, Ph.D. Revised edition. Boston, Ginn & Company, publishers. 1898. Pp. 381.

The appearance of a new member of as popular a series of text-books as Professor Gage's works on physics is an event of considerable importance in educational affairs. It requires a peculiar genius to write an acceptable text-book on a science which involves such considerable inherent difficulties as physics, and the requirements are particularly severe when the book is addressed to the comparatively immature students of our secondary schools. Absolute soundness of scientific statement must be insisted upon, for the student is not in a position to correct, from his own knowledge, misstatements of fact; and to this quality of accuracy must be joined a judicious selection and proper presentation of the subjects treated, for it is upon these qualities that the efficiency of the work as a manual for instruction depends. If this review shall seem to contain many words of censure and few of praise it is because Professor Gage's skill in presenting a subject is well known and needs no comment, while the deficiencies and errors of a book of this character ought not to pass unnoticed.

It is, unfortunately, a fact that in regard to soundness and accuracy of scientific statement this edition is open in places to considerable criticism. We notice that acceleration is measured in *centimeters per second*, the unit of velocity; that[1] "*loudness is not proportiona lto this energy*" (of the vibrating air particles) where the introduction of the word *simply* before proportional would have made an accurate statement of it;[2] that the Edison lamp is made of carbonized bamboo, that an Edison 16-candle-power lamp has a resistance (when hot) of about 140 ohms, the difference of potential at its terminals is about 110 volts and it requires a current of 0.75 ampère. The statement regarding the material of the filament is excusable enough and even the description of a 5.2 watt per-candle lamp is not particularly reprehensible, but surely the author might have made his quantities check out according to Ohm's law and might have intimated that there are other lamps than 110-volt lamps. Moreover, the reason given for exhausting the air from the bulbs of glow-lamps, namely, to prevent the oxidation of the filament, is far from being a sufficient one.

[1] § 181. The italics are Professor Gage's. [2] § 376.

The statement that "*A unit of electrical work is the work done by a unit current in a unit time,*"[1] will scarcely bear searching scrutiny.

A novel problem is proposed.[2] "If you have a dozen quart cells, how can you make them equivalent to one three-gallon cell?" Is this a bit of quiet humor, or is Professor Gage hinting at a newly discovered law in regard to voltaic cells?

It is with the treatment of the theory of the primary battery that the most serious fault can be found. The statement is made:[3] "The greater the disparity between the two elements with reference to the action of liquid on them, the greater the difference in potential; *hence the greater the current.*" The last clause should surely have been omitted.

In a footnote on this same page the statement is made that "electricity always flows from a point of high potential to a point of relatively low potential," and hence, since the current flows from zinc plate, through the electrolyte to the copper plate, the zinc plate is positive with respect to the copper plate, while, at the same time, the "electrode" presumably attached to the copper plate, or forming a part of it, is positive with respect to the "electrode" attached to the zinc plate. In pursuance of this fundamental fallacy the effects of polarization are explained in this wise: "A deposit of hydrogen on the copper raises, in some measure, the potential of this (negative) element and thereby diminishes the potential difference between the two elements." It may well be that it would be impracticable to frame an exact definition of difference of electrical potential which would be within the comprehension of the average pupil of the class which this book is designed to serve, but we surely have a right to expect that the author in his use of the term as applied to generators of electricity, should not make statements which are in contravention of the principle of the conservation of energy.

We hardly think that any one will find fault with the author for any failure or his part to present sufficient matter for a high school course in physics. Indeed, it would seem that some things might have been omitted from the book without impairing its usefulness. Certainly a reference to that esoteric and topsy-turvy unit the "mho" is not indispensable in a work of this character. It is also a serious question as to the advisability of including in the early pages of a beginner's book a statement of the molecular theory of the constitution of matter. This theory is of enormous utility in explaining many natural phenomena, but it would seem that its presentation might at least be delayed until by the study of a goodly number of the phenomena the pupil had come to a position where such explanation or correlation of facts could be in some measure

[1] § 317. The italics are Professor Gage's.
[2] Page 311. Problem 8.
[3] § 298. The entire statement is italicized in the book.

appreciated. The danger is that younger pupils may come to have an altogether too specific opinion as to the exact nature and properties of molecules and a too unshakable faith in the objective accuracy of that opinion.

It is questionable also if the quantity *momentum*, which is so useful in theoretical mechanics after the principle of the conservation of momentum in a closed system has been introduced, should have any place or mention in a book of the character of the one in question. The use of the term leads to no end which cannot be equally well attained without it, and it is a distinct gain to the teacher to be relieved of the necessity of combating the confusion which is sure to arise in the minds of the pupils between momentum and kinetic energy.

The section which, under the caption of "alternating currents," treats very briefly of Tesla's spectacular experiments with alternating currents of high frequency and high potential, might have been omitted without serious loss, especially if the space so gained had been turned to account in the discussion of the static transformer as used in practice. We search in vain in the description of this important instrument for any intimation of the fact that it is used only with alternating currents, while the attempt made in the same paragraph to explain why, in power transmission, it is economical of copper to use high potentials, is absolutely footless.

The treatment of the metric system of units is not all that might be desired. These units and the c. g. s. units in general, are defined, indeed, but apparently little effort is made to enforce the fact that they are really usable, the most usable units that we have. By far the greater part of the problems are founded on the British units. This is, in the reviewer's opinion, a distinct defect in any text-book on physics.

We ought also to mention what seems, superficially at least, to be a case of vicious *argumentum in circulo*. We refer to section 317 in which, *apparently* at least, definitions are made of unit current, unit e. m. f. and unit resistance each in terms of the other two, according to Ohm's law.

While the reviewer has thought it necessary to enumerate a number of specific points in respect to which this work is open to severe criticism, and while the list might be somewhat further extended, he would not for a moment convey the impression that these faults are at all characteristic of the work as a whole. The specific faults can easily be corrected by the careful teacher, and we are not disappointed in supposing that Professor Gage's long experience both as a teacher and as a writer, and the reception which his previous books have been accorded by hundreds of teachers of physics, are a sufficient guarantee for the general excellency of the presentation of the subject-matter and for the adaptability of the book to the teachers' and pupils' use. The method of treatment is

didactic and only to a very limited extent deductive, while the so-called "Inductive Method," now so thoroughly discredited, has been abandoned entirely.

C. H. SHARP.

CORNELL UNIVERSITY.

Les Modifications Permanentes du Verre et le Déplacement du Zéro des Thermomètres. By L. MARCHIS. Pp. 442. Paris, A. Hermann, 1898.

This memoir forms an important addition to our knowledge of the subject of the behavior of glass under fluctuations of temperature, a topic which is not only important on account of its application to thermometry, but likewise in its bearing upon the theories of the properties of matter. In order to exaggerate the change of zero, special thermometers were constructed with very large bulbs. The stems of these instruments were of narrow bore for a considerable range above and below the temperature, which was to constitute the lower of the two fixed degrees between which the fluctuations were to be made. The bore was much enlarged from a point just below the steam point and the stem from 100° upwards was graduated in the centigrade scale. The graduation throughout the lower range was in millimeters. By the introduction of nitrogen into the space above the mercury of some of these thermometers, Marchis was able to extend his experiments up to 440°, the boiling-point of sulphur. Four other upper limits of oscillation were likewise used: 357°, the boiling-point of mercury; 310°, the boiling-point of diphenylamin; 185°, the boiling-point of anilin; and 100°, the boiling-point of water. Certain experiments were carried on in which the oscillations were extended downwards to −80°, that being the temperature obtained in a bath of carbon dioxide and ether.

The zero point was an artificial one fixed arbitrarily at 60° C., at which temperature the thermometers were compared with a standard maintained constantly in a reservoir of that temperature, the heat of which was regulated mechanically with great precision. Each of the thermometers was subjected to rapid fluctuations between this standard zero at 60°, and one of the other temperatures just mentioned a very large number of times, about 1,500 in the case of each instrument. The curves of rise of zero with successive heatings and coolings showed an effect diminishing rapidly as the number of oscillations increases. The sensitiveness of the apparatus may be inferred from the statement that the shifting of the zero amounted in some experiments to about 175 mm.

The author classifies the phenomenon of permanent deformation in glass as a form of *hysteresis*, comparable to that occurring in metals such as zinc and brass, when rapidly stretched and relaxed, and to the mag-

netic hysteresis observed in iron. By an extension of the thermodynamic equations of Duhem, Dr. Marchis succeeds in constructing expressions for these types of deformation in spite of the fact that they are in contradiction with the fundamental hypotheses of the usual thermodynamic equation, namely, the existence of an equation of equilibrium, and the assumption that all modifications which result in states of equilibrium are reversible. By numerous modifications of the experiments which I have just indicated the nature of this hysteresis phenomenon in glass is very completely worked out, and the author builds up a theory by successive approximations, adding variable after variable until he reaches a differential equation of general form containing four such quantities.

The author points out in the closing chapters of his investigations that the theory which he has developed is one which would dictate precisely the treatment of thermometers which has been adopted as the result of experience by the most skillful makers, and by the most successful observers. It is greatly to be regretted that he did not include in the instruments which he subjected to investigation, thermometers made of the Jena glass. The importance of this glass in optics and in all physical work is increasing from year to year, and the exhaustive study of its properties is fully as important as is that of the varieties of glass which are included in these experiments; viz.: the white glass known as *Christal-Guilbert-Martin* and the *verre vèrt*, of which the thermometers of the best French makers are constructed.

E. L. NICHOLS.

Ostwald's Klassiker der exakten Wissenschaften. No. 101. Abhandlungen uber Mechanische Wärmetheorie. By G. KIRCHHOFF, pp. 48; No. 102. Ueber Physikalische Kraftlinien. By JAMES CLARK MAXWELL, pp. 146. Leipzig, Wilhelm Engelmann, 1898.

Numbers 101 and 102 of Ostwald's series of reprints contain respectively Kirchhoff's three papers on the Mechanical Theory of Heat and the translation by Boltzmann of Maxwell's well-known papers on Lines of Force. Nearly half of the latter volume consists of a series of notes by the translator, which will add greatly to the interest with which it will be read. Many who would have preferred to have read Maxwell in the original—and his papers are easily accessible both in the volumes of the Philosophical Magazine and in the reprint of his scientific papers—will welcome Boltzmann's commentary. Kirchhoff's papers which appeared in Poggendorff's Annalen in 1858 are also readily accessible to most men of science, especially in Germany. It is pleasant and convenient, however, to have them gathered together and incorporated in the Ostwald collection.

E. L. N.

Elements of Physics. By E. L. NICHOLS and W. S. FRANKLIN. Vol. I., Mechanics and Heat. New York, The Macmillan Co., 1898. Pp. vi + 219.

We have received from the publishers a copy of the second edition of this work, which has been completely revised and in a large part rewritten.

The American Annual of Photography and Photographic Times Almanac for 1899. New York, The Scovill & Adams Company. Pp. 370 [Received.]